Modelling the Naira Exchange Rate Dependence Using Static and Time-Varying Copula

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This paper examines the dependence structure of different currencies versus the Nigerian Naira using constant and time-varying copula. Daily Naira/USD, Naira/Yuan, Naira/Pound, and Naira/Euro exchange rates from 23 December 2011 to 12 May 2020 were utilised. We fitted eight constant and time-varying copula families using the exchange rate standardised residuals. The study finds that the Naira exchange rate may be estimated with student t-copula, Symmetrized Joe-Clayton (SJC), or Rotated Gumbel copula models and Autoregressive (AR)– Glosten Jagannathan Runkle-Generalized Autoregressive Conditional Heteroscedastic (GJR-GARCH) (1,1) models with skewed t residuals for margins. The Naira exchange rate returns is time-varying, tail-dependent, and asymmetric. The study recommends that portfolio diversification, asset allocation of Central Bank of Nigeria foreign reserves, bank risk capital aggregation, and risk management decisions should not be based on linear correlation coefficient (Gaussian copula) but on copula models that can capture asymmetry and tail dependence, such as Student t, SJC, and Gumbel copulas.

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1. Introduction

Foreign exchange is necessary for modern trade and economic development. The dependence of exchange rates has been recognized as vital for its link to numerous important areas, such as the coordination of economic policy assessment for international policy, management of risk, asset allocation and the integration assessment of different economies, testing the efficiency of financial markets and contagion assessment of financial markets. A central bank should be interested in understanding foreign exchange dependence and its possible asymmetry for at least two reasons.

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First, knowledge about the comovement of a currency against foreign currencies can serve as guidance on a domestic economy's response to competitiveness and change in aggregate import prices. Second, with the knowledge of the comovement of exchange rates, particularly if it involves currency interventions, then it can strive to attain some level of appreciation/depreciation against one or more foreign currencies (Scotti & Benediktsdottir, 2009).

The analysis of exchange rate dependency and potential asymmetry is also crucial for other reasons. Exchange rate fluctuations can impact the economy and inflation by affecting import pricing and export competitiveness. Understanding how the dollar fluctuates in relation to other currencies might impact Naira competitiveness and import pricing. In addition, central banks benefit from understanding how exchange rates interact, particularly during currency interventions if they want to achieve a specific degree of appreciation/depreciation versus multiple foreign currencies. Furthermore, valuing derivatives like multivariate currency options, which hedge against exposure to several currencies, requires consideration of exchange rate correlation and tail dependence. Failure to recognise potential asymmetric reliance might lead to inaccurate asset valuations (Scotti & Benediktsdottir, 2009; Salmon & Schleicher, 2006; Erdemlioglu *et al.*, 2013).

It is well-known that a multivariate model has two parts: the marginal or univariate part, which defines financial variables, and the dependent structure, which describes how the variables are joined. However, since the work of Mills (1927), it is recognized that the univariate or marginal distributions of most financial time series are not normally distributed, they are skewed and possess fat-tails. The two main approaches for modelling dependence are either to assume independence or multivariate normality of the random variables. While these two assumptions are straightforward to implement, they have been shown to misrepresent reality, at least for most financial time series like those obtained from the foreign exchange market (Dias, 2004). When multivariate normality is assumed for the random variables, Pearson or linear correlation is used as the measure of dependence between the variables.

The presence of non-linearity in financial time series invalidates the Gaussian distribution assumption that supports the use of Pearson correlation that relates returns or factors in a linear manner. Also, the characteristics of high skewness and heavy-tailedness of financial data, further nullify using Pearson correlation as the ideal measure of dependence between variables. This is because Pearson correlation only supports low probability of experiencing extreme observation. In addition, since linear correlation is believed to be constant across time, it is unable to account for the asymmetry in asset returns (Ang & Chen, 2002; Longin & Solnik, 2001). Asymmetric dependence exists, for instance, when returns move together more frequently during market downturns than during booms. The only situation where Pearson or linear correlation is appropriate as a dependence measure is when the distributions are multivariate spherical and elliptical, such as multivariate normal distributions and multivariate t-distributions (Patton, 2001; McNeil *et al.,* 2015).

Given the relevance of modelling dependence for exchange rate data, coupled with the inadequacy of Pearson correlation as a dependence measure to incorporate asymmetry and non-normality, other dependence measures used to model dependence between exchange rates includes copula and the associated rank correlation measures (McNeil *et al.,* 2015). Dias (2004), Scotti & Benediktsdottir (2009) and Patton (2006), all investigated time-varying and constant dependence for different exchange rate series. Loaiza *et al.* (2015) used copula to evaluate interdependence between the exchange rates of Brazil, Argentina, Colombia, Chile, Mexico and Peru.

More recently, Katata (2020) investigated dependence constant and time-varying copula models, Terzi *et al.* (2022) modelled the dependency structure between variables in a comprehensive and dynamic way, while Gong *et al.* (2022) used the mixed-frequency copula model to examine how economic fundamentals affect exchange rate dependence using the Canadian dollar, British pound, and Japanese yen.

The Nigerian foreign exchange market is crucial for the economy. As a net exporter of crude oil, accurately modelling and forecasting of exchange rate between Nigeria's and its trading partners is essential. Consequently, the Naira exchange rate has been extensively studied (Mordi,2006; Katata, 2016; Isenah & Olubusoye, 2016). However, there are no previous studies on Naira exchange rate dependence and its potential asymmetry using copula.

The aim of this paper is to fill this gap by analysing Naira exchange rates co-movement and dependence against USD, Euro, Yuan and Pounds exchange rates using constant and time-varying copula. The paper also examines asymmetry in the dynamic dependence between the Naira against USD, Euro, Yuan and Pounds exchange rates. This paper captures the constant and dynamic behaviour of dependence between the currencies that allows the copula parameters to change over time. To the best of our knowledge, this is the first paper to investigate Naira exchange rate dependence using copulas. Furthermore, copulas allow for an alternative view on dependencies beyond multivariate normality which this paper explores.

The rest of the paper is structured as follows: literature review is presented in Section 2 while the methodology is presented in Section 3. The results and discussion are presented in Section 4; finally, the conclusion and policy recommendations are provided in Section 5.

2. Literature Review

In this section, the paper presents the theory of multivariate modelling using copula and briefly reviews the empirical literature on modelling dependence with copula.

2.1 Theoretical Literature

A multivariate model consists of two parts. They are the univariate or marginal part that describes every variable and the second part, the dependence arrangement between the univariate variables. Since the early 1960s, according to Mandelbrot (1963), asset return distributions have been shown not to be normal due to excess kurtosis and skewness that are larger than those found in a normal distribution.

In general, the linear correlation coefficient was utilized as a measure of dependence for the dependence structure. While most real-world data distributions are not from spherical and elliptical distributions, linear correlation is the dependence measure strictly for these distributions (Embrechts *et al.,* 2002; Joe, 2014). Being invariant under strictly increasing linear transformations is a desirable quality of linear correlation as a dependence measure. Since it is not invariant under nonlinear increasing transformations, linear correlation is not a measure of concordance (Embrechts *et al.,* 2003).

Copula functions, often known as copula, are excellent tool for modeling dependence. Other dependence measures and concepts that are used for multivariate non-Gaussian distributions and copula include: Frechet bounds, the concordance ordering, measures of monotone association (like Kendall's tau, Spearman's rho, Blomqvist's beta), positive quadrant dependence, and tail dependence (Joe, 2014).

Copula theory applications rely on the Sklar theorem, which allows the decomposition of an n-dimensional joint distribution into its n univariate distributions, and a copula function, which determines the dependent arrangement among the variables (Cherubini, *et al.*, 2004; Nelsen, 2006; Palaro & Hotta, 2006).

Copulas are functions that define the dependence between two or more random variables. Sklar's theorem enables the mapping from the separate distribution functions to the joint distribution function. Because of Sklar's theorem, the marginal distributions do not have to be identical to one another, nor does the choice of copula have to be particular to the marginal distributions. Kendall's tau, Spearman's rho and tail dependence coefficients are measures of dependence that are scale invariant, unaffected by strictly increasing transformations of the underlying variables, can be expressed as a function of the copula alone. Commonly used parametric copulas for financial applications include the Gaussian or normal copula, Student t copula, the Frank copula, the Gumbel copula, and the Clayton copula. Copulas can be divided into conditional and unconditional. The conditional copula is to model dependencies that change over time or are not constant (Fan & Patton, 2014).

Fan & Patton (2014) and Patton (2006) discuss two methods for estimating copula models and their parameters: full maximum likelihood and two-stage inference functions for margins (IFM). In the first, marginals are identified and estimated, whereas in the second, copula functions are identified, estimated, and analysed.

In this paper, the IFM method for estimation of copula functions is used to model the dependence of Naira against four other currencies, as carried out by Dias (2004) and Patton (2006). To study the different foreign exchange rates dependence, there is a need to specify both the marginal models for the exchange rate returns (first step of the IFM) as well as a joint model for the dependence using copula models (second step of the IFM).

Note that due to the possible heteroscedastic behaviour of the exchange rate return, we will not only apply copula functions to the observed returns directly. Copula functions will be estimated using the innovations after fitting univariate (marginal) AR-GARCH models to the individual return series of Nigerian exchange rates. This approach has been successfully used by Dias (2004) and Patton (2006).

We therefore specify marginal models for the Naira against USD, Euro, Yuan and Pounds exchange rates by allowing each series to have conditional mean and variance that both vary with time. It is well-known that with parametric models for the mean, variance and error distribution, the conditional distribution of a univariate series has been completely characterized.

2.2 Empirical Literature

Katata (2016) showed that the return series for the Naira exchange rate demonstrated time-varying volatility, serial correlation, as well as fat tails. Also, because of the presence of asymmetry observed in the Naira exchange rate return distributions, the study suggested that it is essential to model both left and right tails separately to capture their dissimilar features.

Dias (2004) studied copula models for financial market for both conditional and unconditional copulas. For a total of 649 observations based on weekly frequency from April 28, 1986 through October 4, 1998, the study used bivariate tick-by-tick foreign exchange spot rates for the US dollar against the German mark (USD/DEM) and against the Japanese yen (USD/JPY). Student t distributions was used to model each univariate return time series. The study concludes that the weekly returns on the USD/JPY and USD/DEM spot exchange rates can be described by a combination of the Gumbel and Rotated Gumbel copula or by the student t-copula, along with univariate t-distributions for the margins, from the analysis of the unconditional and static iid (under the assumption that the return vectors are iid).

Based on the assumption that the return vectors are not iid, Dias also investigated the association between the residual vector parts after fitting a dynamic model. In the first instance, the data is filtered through ARMA-GARCH models and the residuals analysed with copulas. From this analysis for the conditional and static iid, Dias concluded that the filtered residuals on USD/DEM and USD/JPY spot rates can be modelled well by the t-model or by a mixture between the Gumbel and survival Gumbel copula. In Dias (2004), a time-varying copula was also used together with two univariate ARMA-GARCH models. The fitting of GARCH-type models to the exchange rate returns revealed evidence of time variation of the dependence arrangement. This was done using a copula function for the bivariate residuals linking two ARMA GARCH models. Finally, Dias (2004) fitted student t univariate innovations and student t-copula to the exchange rate returns and observed that the degrees of freedom of the univariate distributions are much lower than those of the copula, implying that the univariate distributions of the exchange rate returns have fatter tails than the dependence system that binds them.

Patton (2006) applied copula to describe the time-varying dependence of the DM–USD and Yen-USD exchange rate covering January 1991 to December 2001. The univariate distribution for the DM–USD exchange rate is presumed to be characterized by AR(1), and student t-GARCH(1,1), while the univariate distribution for the Yen–USD exchange rate is presumed to be described by an AR(1), student t-GARCH(1,1). Because they will be combined with the copula models for the dependence structure, Patton referred to the marginal distribution specifications as the ”copula models” for the marginal distributions. The copula models used to characterize the dependence between the exchange rates are the “symmetrized Joe–Clayton” copula and the Gaussian copula, for both constant and time-varying versions. The study found evidence that the dependence amongst these exchange rates is not constant but varies with time. The study also found evidence of symmetry, such that dependence is larger during rises of the USD versus DM and the yen than in periods of declining USD.

Using data from June 2005 to April 2012, Loaiza *et al.* (2015) assessed the degree of contagion between the currency rates of Brazil, Argentina, Chile, Mexico, Colombia, and Peru. They measured contagion based on tail dependence coefficients where contagion is viewed as interdependence. Their findings indicated that these nations can be divided into two blocks. The first block includes Brazil, Colombia, Chile, and Mexico, whose exchange rates displayed the highest dependence coefficients. The second block includes Argentina and Peru, whose dependence coefficients on exchange rates were relatively low when compared to those of other Latin American nations. The study found that majority of Latin America's exchange rate pairs displayed asymmetric behavior with far less tail dependency.

Cubillos-Rocha *et al.* (2019) used a sample from April 3 2006 to February 15 2018 to study exchange rate dependence for seven countries that includes two developed countries, the United Kingdom and Germany, South Korea and Indonesia as two large emerging Asian economies, Brazil, Chile, and South Africa. The authors created a multivariate copula using a normal vine copula, which allows for an extremely flexible dependence structure. The study revealed evidence of contagion between the exchange rate pairs of these countries as well as asymmetric relationship between the pairs. It was discovered that currency contagion only happens when the value of the other currencies relative to the USD increases. The pair made up of the currency rates of Germany or the UK also has the highest tail dependence coefficient. Furthermore, there is evidence that contagion arises more between countries within the same region.

Nguyen and Huynh (2015) applied conditional copula to estimate value-at-risk of a portfolio composed of 6 currencies to that of Vietnam. The study modelled the marginal return series of each currency pair using AR(1)-GARCH (1,1) model and the residuals were concurrently produced using Normal and t copula. The data used has 1328 daily closing prices, from 2^{nd} January, 2007 to 30^{th} March, 2012. The study concluded that all models are quite stable.

Dias & Embrechts (2010) proposed a time-varying copula-GARCH model to describe the evolving dependence between Euro/USD and Yen/USD exchange rates. The study used spot exchange rates for the US dollar, the euro, and the Yen from October 1, 2000 to October 1, 2008, at intervals of sixty minutes. The results revealed a considerable correlation that varies with time and depend on the historical return realizations. The proposed time-varying copula specification outperformed other dynamic benchmark models like the BEKK model. The univariate distributions of the Euro/USD and Yen/USD returns were well modelled by a t distribution and the t-

copula performs best for the static models to describe the dependence between the return series while the Gaussian and *t*-copula models emerged as the best models to describe time-varying dependence. The study documented the time-varying nature of the exchange rate dependence and the superior performance of the time-varying copula models (Gaussian and t-copula) over their time-invariant counterparts.

Katata (2020) used Pearson linear, rank correlations and copula functions to study contagion in West African Monetary Zone (WAMZ) exchange rates versus the USD from 1995 to 2020. The author examined dependence structures using several constant and time-varying copula models. Following the global financial crisis of 2007–2009 and the COVID19 pandemic, the results showed significant changes in reliance levels, tail behavior, and asymmetry patterns across exchange rate returns. There was also evidence of increased dependence among the majority of currencies.

Mili & Bouteska (2023) predicted correlations between cryptocurrency and major fiat currencies utilising Generalised Autoregressive Score (GAS) time-varying copulas. The authors modelled tail dependence between conventional currencies and Bitcoin using a GJR-GARCH-GAS copula specification and found that static models cannot fully reflect Bitcoin's severe dependency on fiat currencies.

Hu *et al.* (2023) used the dynamic conditional correlation multivariate volatility Copula and BEKK models to analyse the dynamic dependence relation and volatility spillover effect between US dollars and Chinese RMB exchange rates using offshore and onshore market data. They showed a positive time-varying dependence link between the offshore and onshore RMB exchange rates, with a gradual price convergence. They found a positive and statistically significant time-varying dependence between the RMB exchange rate in the offshore and onshore markets, with a gradual price convergence.

Djemo (2022) quantified foreign exchange rate risk and modelled the dependence structure of underlying assets with selected stock. The author found that the GJR-GARCH with Student's t-distribution combined with a regular-vine (R-vine) copula outperforms the alternative models.

To the best of our knowledge, there are no previous studies on Naira exchange rate dependence and its potential asymmetry using copula. This paper therefore, studies Naira exchange rates co-movement and dependence against USD, Euro, Yuan and Pounds exchange rates using constant and time-varying copula. Specifically, we fit Gaussian, Student t, Gumbel, Rotated (survival) Gumbel, Clayton and SJC constant and time-varying copulas to the standardized residuals on the four exchange rate series.

3. Data and Methodology

3.1 Data

The data used for this study covers the period 23^{rd} December, 2011 to 12^{th} May 2020, yielding 2067 observations. The data were obtained from CBN Statistical Database. These exchange rates (i.e. USD, Yuan, Pound and Euro with respect to the Nigerian naira) are selected because they represent the most traded currencies in Nigeria. The study made use of exchange rate returns in the analysis.

Let P_t be the closing price on ith day and r_t is the continuously compounded return on day t. Then the daily return of Naira against USD, Euro, Yuan and Pounds exchange rates can be obtained as $r_t = \log(\mathrm{P}_t / \mathrm{P}_{t-1})$.

3.2 Models for the Marginal Distribution

It is well-known that exchange rate series have variance that evolves with time, possess long-memory as well as fat tails (Katata, 2016; Patton, 2006). The GARCH models, specifically GJR-GARCH is used to describe the univariate distributions of the exchange rate return series.

Let μ be the expected return and ε_t a zero-mean white noise. Then the mean equation is specified as :

$$r_t = \mu + \varepsilon_t \tag{1}$$

$$\varepsilon_t = h_t e_t \tag{2}$$

The evolution of the GJR-GARCH(P,O,Q) process for conditional variance h_t is defined as:

$$h_t^2 = \omega + \sum_{p=1}^{P} \alpha_p \varepsilon_{t-p}^2 + \sum_{o=1}^{O} \gamma_o \varepsilon_{t-o}^2 I_{[\varepsilon_{t-o}<0]} + \sum_{q=1}^{Q} \beta_q h_{t-q}^2 \tag{3}$$

where μ_t can be an adapted model for the conditional mean and $I[\varepsilon_{t-o}<0]$ is an indicator function that becomes 1 if $\varepsilon_{t-o} < 0$ and 0 otherwise. The conditional variance equation contains the ARCH terms ε_{t-p}^2, the GARCH terms h_{t-p} and the GJR terms $\varepsilon_{t-o}^2 I_{[\varepsilon_{t-o}<0]}$. Other distributions for the error term include student t, skewed student and Generalized Error Distributions, and may involve more parameters as well as restrictions. The error term ε_t of equation (1) is presumed to have a conditional variance h_t that is time-varying as specified in (3).

All the parameters (ω, α, γ, β) are simultaneously estimated by maximizing the log likelihood for the four exchange rate series. It should be noted that when the distribution of the errors is *Student t, then an* extra parameter (ν), is required, while additional ν and λ parameters are required for Hansen's Skew-t density. The Hansen skewed student *t* distribution has two "shape" parameters. The first is a skewness parameter, $\lambda \in (0; 1)$ that controls the extent of asymmetry, while the second parameter is the degrees of freedom $\nu \in (2; \infty]$ that controls the thickness of the tails. When $\lambda = 0$, the standardized Student's *t* distribution is recovered, when $\nu \to \infty$, the skewed Normal distribution is obtained, and when $\nu \to \infty$ and $\lambda = 0$, then the N (0; 1) distribution is obtained. While student t distribution can capture excess kurtosis, a skewed student t can capture both skewness and kurtosis.

3.3 Measures of Dependence

The measures of dependence used in this paper are linear or Pearson's correlation and rank correlation measures (Spearman's rho and Kendall's tau). The linear correlation coefficient between random two variables X and Y is equal to the covariance of the two random variables divided by the product of their standard deviations, given as:

$$\rho(X,Y) = \frac{COV(X,Y)}{\sqrt{V(X)V(Y)}} \tag{4}$$

where for *V(X)* and V(Y) are the variances of X and Y, respectively.

Let ρ be Pearson 'linear' correlation, X and Y be random variables with distribution functions F_1 and F_2 and joint distribution function F. Spearman's rank correlation is defined as

$$\rho_S(X,Y) = \rho(F_1(X);F_2(Y)) \tag{5}$$

For a sample with n observations (x_i,y_i) and i=1...n, then the Kendall's tau, τ, is obtained by associating all the likely sets of observations{(x_i,y_i),(x_j,y_j)} for $i \neq j$.
The empirical form of Kendall's τ is given by the following expression, where P_n and Q_n are the number of concordant and discordant pairs, respectively, for a sample of n observations.

$$\tau = \frac{P_n - Q_n}{\binom{n}{2}} = \frac{4}{n(n-1)} P_n - 1 \tag{6}$$

3.4 Copula Models

According to the Sklar's theorem, every multivariate distribution with continuous marginals has a distinct copula form, which shows that if a function C: $[0,1]^n \rightarrow [0,1]$ meets certain regularity requirements, it implies a copula. Simply put, a copula is a multivariate distribution function with uniformly standardized univariate margins.
The bivariate copula is as follows: Let H be joint distribution function with margins F and G, then there exists a copula C: $[0,1]^2 \rightarrow [0, 1]$, for all $(x, y) \epsilon R^2$, such that

$$H(x,y) = C(F(x),G(y)) \tag{7}$$

This statement implies that the joint distribution (H) in n dimensions could bebroken down into the equivalent n univariate marginal distributions (F & G) and copula function (C: $[0,1]^n \rightarrow [0, 1]$).
The families of copula utilised in this paper are the constant and time-varying Gaussian, Student t, Clayton, Gumbel and Symmetrized Joe–Clayton. These are also some of the well-known copula families.

3.4.1 Gaussian (Normal) copula

Let $\Phi(.)$ be the normal distribution, Φ^{-1} the inverse normal distribution and $\Phi_{\rho}(.)$ the bivariate normal distribution with correlation coefficient ρ restricted to [-1,1]. The normal copula, is elliptical copula, defined as:

$$C(u,v) = \Phi_{\rho}(\Phi^{-1}(u),(\Phi^{-1}(u_v)) \tag{8}$$

Both positive and negative dependence can be modeled using the Gaussian copula. This is the most frequently applied copula for financial applications. Even for marginal distributions that are not Gaussian, a Gaussian-type of dependence can be preserved using the Gaussian copula. The Gaussian copula's suggested dependence structure is radially symmetric and free of tail dependence.

3.4.2 The Student t Copula

The Student t distribution, with parameter υ indicating the degrees of freedom and accounting for creating the fat tails, is used in place of the normal distribution in univariate models in the elliptical universe to capture fat tails. Despite having tail dependency, the student t copula is symmetric, like the Gaussian copula. Therefore, assets that appear to be evenly distributed can be "connected together" utilizing student t copula and may be simultaneously exposed to extremely high or low movements. However, the dependence coefficients of the lower and upper tail are equal because of the radial symmetry of elliptical distributions. Similar to the Gaussian copula, this copula uses the Pearson linear correlation coefficient as its dependence measure.

Let d be the dimension of a copula, t_{υ} the distribution function of a standard univariate t distribution and $t_{\upsilon,P}$ the joint distribution function (df) of the vector $X \sim t_d(\upsilon,0,P)$ with P as a correlation matrix. The d-dimensional t copula is given as:

$$C^t_{\upsilon}p(u_1,\ldots,u_d) = t_{\upsilon,\,p}(t_{\upsilon}^{-1}(u_1),\ldots,t_{\upsilon}^{-1}(u_d)) \tag{9}$$

Both the Gaussian and Student t copulas belong to the family of elliptical copulas which are obtained directly from the class of elliptical distributions.

3.4.3 Clayton copula

Let *u =F(x), v = G(y),* and θ be the dependence parameter restricted on the region (0,∞). The Clayton copula takes the form:

$$C(u,v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta} \tag{10}$$

Clayton copula is asymmetric and shows lower tail dependence.

3.4.4 Gumbel copula

For $1 \leq \theta < \infty$, the bivariate Gumbel copula is given as

$$CG(u,v) = exp(-((-\ln u)^{\theta} + ((-\ln v)^{\theta})^{1/\theta} \tag{11}$$

The Gumbel copula is limited to positive dependence only and like Clayton, does not permit negative dependence but shows strong right tail dependence and relatively weak left tail dependence. It has only upper tail dependence but no dependence in the lower tail.

3.4.5 Rotated Gumbel Copula

Unlike Gumbel, this copula possesses only lower tail dependence. For $1 \leq \theta < \infty$, the bivariate Gumbel copula is given as:

$$C(u,v) = u + v - 1 + CG(1-u, 1-v) \tag{12}$$

3.4.6 The Symmetrized Joe–Clayton copula

Let $k = \frac{1}{log_2(2-\tau^U)}, \gamma = \frac{-1}{log_2(\tau^L)}$ and $\tau^i \in (0,1)$. The Joe–Clayton copula is given as:

$$C_{JC}\left(u, v|\tau^U, \tau^L\right) = 1 - \left(1 - \left[1-(1-u)^k\right]^{-\gamma} + \left[1-(1-v)^k\right]^{-\gamma} - 1^{-1/\gamma}\right)^{1/k} \tag{13}$$

The original Joe-Clayton (JC) copula, also called BB7, has asymmetry when there is quality in the two tails of the dependence measures. Patton (2006) proposed the "Symmetrized Joe–Clayton" copula (SJC) as the solution to the issue as follows:

$$C_{SJC}\left(u, v|\tau^U, \tau^L\right) = \frac{1}{2}\left(C_{JC}\left(u, v|\tau^U, \tau^L\right) + C_{JC}\left(1-u, 1-v|\tau^U, \tau^L\right) + u + v - 1\right) \tag{14}$$

This copula is a flexible two-parameter copula that captures both lower and upper tail dependence. The two dependence parameters (τ^U, τ^L) range freely and are independent of each other.

The time-varying versions of the above copula are given as follows: Let $\Lambda(x) = (1-e^{-x})/(1+e^{-x})$ be the modified logistic transformation so that ρ_t is restricted to [-1,1]. Also, the correlation coefficient ρ_t follows a time-varying process such that

$$\rho_t = \Lambda\left(\omega + \beta\rho_{t-1} + \alpha\left|u_{i,t-1} - v_{i,t-1}\right|\right)$$

Then

$$C(u_{i,t}, v_{i,t}) = \int_{-\infty}^{\emptyset^{-1}(u_{i,t})} \int_{-\infty}^{\emptyset^{-1}(v_{i,t})} \frac{1}{2\pi\sqrt{(1-\rho_t^2)}} exp\left\{\frac{y_{i,t}^2 - 2\rho_t y_{i,t} z_{i,t} + z_{i,t}^2}{2(1-\rho_t^2)}\right\} dy_{i,t} dz_{i,t} \quad (15)$$

3.4.7 Time-Varying Student t Copula

As specified in time-varying Normal copula, let $\Lambda(x) = (1-e^{-x})/(1+e^{-x})$ be the modified logistic transformation so that ρ_t is restricted to [-1,1] and is given as

$$\rho_t = \Lambda(\omega + \beta\rho_{t-1} + \alpha\left|u_{i,t-1} - v_{i,t-1}\right|)$$

For υ degrees of freedom, let the univariate Student distribution have the following distribution function $t_{\upsilon,\rho_t}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} \frac{\Gamma(\frac{\upsilon+1}{2})}{\Gamma(\frac{\upsilon}{2})\sqrt{(1-\rho_t^2)}} \left(1 + \frac{s^2 2\rho st + t^2}{\upsilon(1-\rho_t^2)}\right)^{-\frac{\upsilon+1}{2}} dsdt$

Then

$$C(u_{i,t}, \upsilon_{i,t}) = \int_{-\infty}^{t_\upsilon - 1(u_{i,t})} \int_{-\infty}^{t_\upsilon - 1(\upsilon_{i,t})} \frac{\Gamma(\frac{\upsilon+1}{2})}{\Gamma(\frac{\upsilon}{2})\sqrt{(1-\rho_t^2)}} \left(1 + \frac{s^2 2\rho st + t^2}{\upsilon(1-\rho_t^2)}\right)^{-\frac{\upsilon+1}{2}} dsdt \quad (16)$$

3.4.7 Time-Varying Gumbel Copula

Let the dynamic process for the time-varying dependence parameter be defined as:

$$\theta_t = \Lambda(\omega + \beta\theta_{t-1} + \alpha\left|u_{i,t-1} - v_{i,t-1}\right|)$$

and $\Lambda(\mathrm{x}) = (1 - e^{-x})^{-1}$

Then the time-varying version of the Gumbel copula is stated as

$$C(u_{i,t}, v_{i,t}) = \exp\{-\left((-log u_{i,t}^{\theta_t}) + (-log v_{i,t}^{\theta_t}\right)\}^{-1/\theta_t} \; for \; \theta_t \geq 1 \tag{17}$$

3.4.8 Time varying Clayton Copula

Let the time-varying dependence processes for this copula be defined as:

$\theta_t = \Lambda(\omega + \beta\theta_{t-1} + \alpha\left|u_{i,t-1} - v_{i,t-1}\right|)$ then

$$C(u_{i,t}, v_{i,t}) = (u_{i,t}^{\theta_t} - v_{i,t}^{\theta_t} - 1)^{-1/\theta_t} \tag{18}$$

3.4.9 Time-varying Symmetrised Joe-Clayton copula

The time-varying SJC copula allows for changing degrees of asymmetry and level of dependence. It is defined as:

$$C(u_{i,t}, v_{i,t}) = 0.5\left(C_{JC}(u_{i,t}, v_{i,t}) + C_{JC}(1 - u_{i,t}, 1 - v_{i,t}) + u_{i,t} + v_{i,t} - 1\right) \tag{19}$$

where the Joe-Clayton copula, CJC is given as:

$$C(u_{i,t}, v_{i,t}) = 1 - \left(1 - [1 - (1 - u_{i,t})^{a_t}]^{b_t} + [1 - (1 - v_{i,t})^{a_t}]^{b_t} - 1^{\frac{-1}{b_t}}\right)^{\frac{-1}{a_t}} \tag{20}$$

and $a_t = \frac{1}{Log(2 - \tau_t^U)}$ and $b_t = \frac{1}{Log(\tau_t^L)}$, τ_t^U, τ_t^L $\in (0, 1)$

Let $\Lambda(\mathrm{x}) = (1 - e^{-x})^{-1}$, the time dynamics equations of the parameters are given as:

$$\tau_t^U = \Lambda(\omega_U + \beta_U \tau_{t-1}^U + \alpha\left|u_{i,t-1} - v_{i,t-1}\right|)$$

$$\tau_t^L = \Lambda(\omega_L + \beta_L \tau_{t-1}^L + \alpha\left|u_{i,t-1} - v_{i,t-1}\right|)$$

The Gaussian (normal) copula has 0 tail dependence for correlation less than one, implying that in the extreme tails of the distribution the variables are independent. The student t copula is symmetric like Gaussian copula, but unlike Gaussian copula, it has tail dependence. Clayton copula can capture asymmetry and shows lower tail dependence. Gumbel copula has only upper tail dependence while "Rotated Gumbel" Copula has only lower tail dependence. The SJC is a flexible two-parameter copula that captures both lower and upper tail dependence.

Therefore, it is important to appreciate the notion of tail dependence, which captures the behavior of the random variables during extreme events. The fat-tail phenomenon in univariate setting is called tail dependence in a multivariate framework. For instance, it measures the probability that we will observe an extremely large appreciation (depreciation) of the Naira against the USD or any other currency, given that the Euro Pound has had an extremely large appreciation (depreciation) against the USD.

4. Results and Discussion

4.1 Descriptive Statistics

Table 1 shows descriptive statistics. The measures of central tendency, arithmetic mean and median, for all the exchange rate returns are very close to zero. That implies that the usual assumption of the random walk model that the expected value of daily returns equals zero is met.

Table 1: Descriptive Statistics of the Naira exchange rate returns

	Naira/Yuan	Naira/Euro	Naira/USD	Naira/Pounds
Minimum	-0.353	-0.359	-0.352	-0.377
Maximum	0.062	0.154	0.054	0.089
Skewness	-23.803	-15.083	-28.626	-17.390
Kurtosis	740.832	431.878	949.879	498.329
Standard deviation	0.010	0.012	0.010	0.012
Mean	0.000	0.000	0.000	0.000
Median	0.000	0.000	0.000	0.000

4.2 Pre-estimation Tests

The sample in Figure 1 displays the behavior of the Naira against USD, Euro, Yuan and Pounds exchange rates over time. All the naira exchange rates exhibit a depreciation drift over the sample period. Figure 1 shows that the four nominal exchange rates have stochastic trend and are therefore nonstationary. The lack of stationarity and normality observed in the Naira exchange rates has been well-documented by Katata (2016). The volatility of the nominal exchange rates is also evident from the figures and spikes coinciding with policy decisions by the CBN that affect the exchange rates.

Augmented Dickey-Fuller test results (not reported in the table) show that the returns are stationary. All the four Naira exchange rates showed negative skewness with the value not equal to zero. This is an indication of asymmetry in the exchange rate distributions. Additionally, all the exchange rate returns showed substantial evidence of fat tails as the kurtosis of the returns is over 3. This is an indication of a considerable violation of normality and the returns distributions are statistically different from those of a normal distribution. As a result of the presence of asymmetry, we have to model the tails, individually so as to capture their different features.

As expected, and observed in Figure 2 (panels 1-4), all series of the returns appear to be mean reverting and exhibit periods of low volatility followed by periods of much higher volatility (volatility clustering).

Figure 2 shows that the four exchange rate series are fairly stationary. The Jarque–Bera test of the normality of the unconditional distribution of the four exchange rates are significant at 5% strongly reject unconditional normality. The stylized facts of the exchange rates series therefore indicate that, their unconditional distributions displayed asymmetry, fat-tailedness and non-normality.

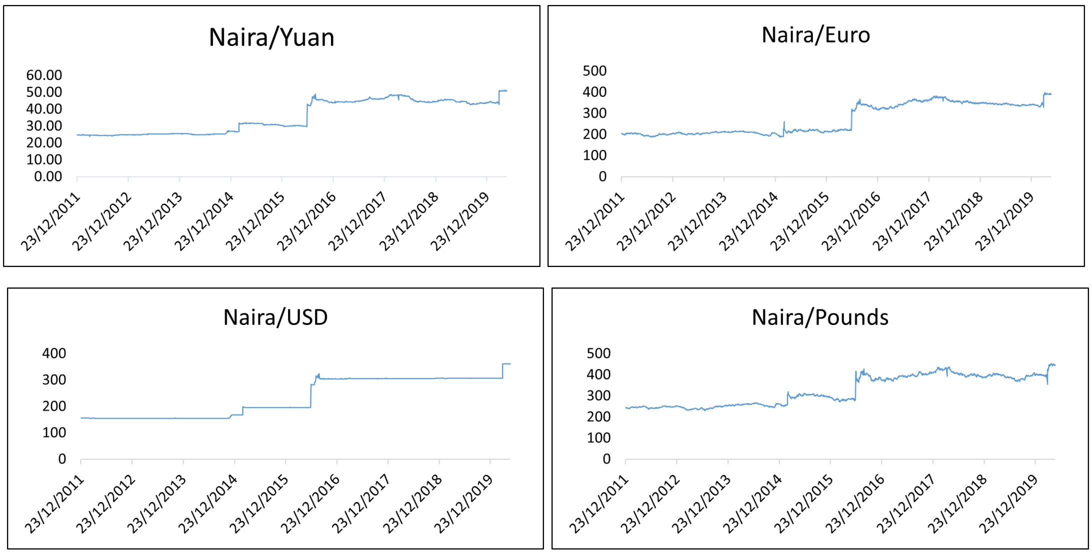

Figure 1: Naira exchange rate behaviour

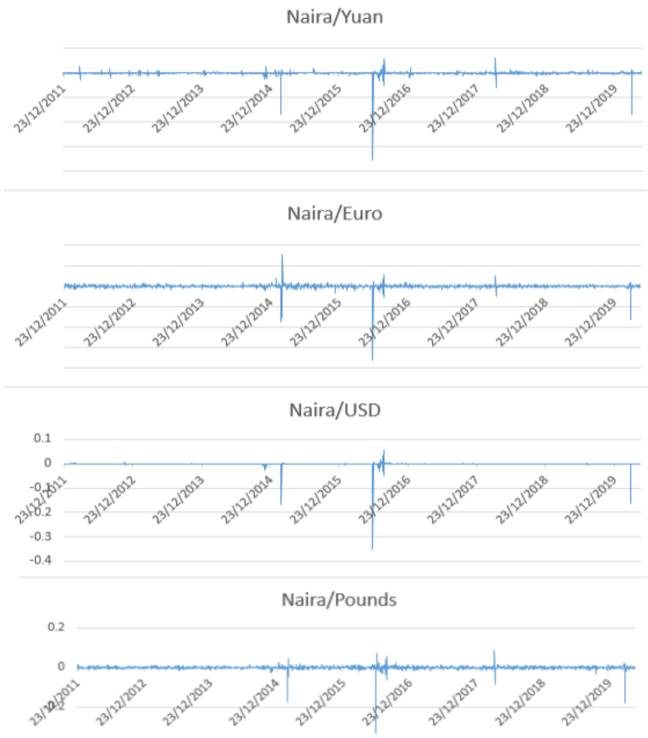

Figure 2: Daily Logarithmic Returns of Naira exchange rates

4.3 Estimation Results

4.3.1 Results of Marginal Distributions

Table 2 shows the optimal models for the exchange rates obtained using both the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) by considering ARMA models for the conditional mean of up to order (5; 5).

Table 2: Conditional Mean Model

Exchange Rate Pairs	Optimal ARMA Model (AIC)	Optimal ARMA Model (BIC)
Naira/Yuan	AR (0), a constant	AR (0), a constant
Naira/Euro	AR (0), a constant	AR (0), a constant
Naira/USD	AR (0), a constant	AR (0), a constant
Naira/Pound	AR (3,3), a constant	AR (0), a constant

Table 2 shows that a constant (AR(0)) model was found to be adequate for the conditional mean of the four exchange rates. For the conditional variance, we use the log-likelihood in Table 3 to select the optimal volatility models in the GJR-GARCH family (Glosten, *et al.*, 1993), of order (1,1) based on normal, standardized student t and Hansen's skewed-t errors as carried out by Patton (2001).

In Table 3, LL stands for log-likelihood. The GJR-GARCH(1,1) model with Hansen's skewed *t* distribution for the errors is selected for each of the four rates given the highest log-likelihood estimates over the estimates from Student-t and normal distributed errors.

A fully parametric model for the four exchange rates therefore contains the AR–GJR-GARCH (1,1) models together with Hansen's skewed *t* distribution for the univariate distributions. After selecting a model, we proceed to the next stage of the Inference for Margins (IFM) method of fitting copula families to the pseudo-observations. However, before fitting copula, we examine the degree of dependence using Pearson correlation coefficient r, Kendall's τ and Spearmann's ρ).

The estimates are presented in Table 4 with p-values in brackets. Because the p-values are lower than the significance level of 0.05, the estimated correlation measures all are significantly different from zero. It should be noted that Kendall's rank correlation coefficient is related to a method for estimating the parameters of some copulas (McNeil, et al., 2015).

From Table 4, Pearson correlation coefficient produced the highest estimate for the Naira/Yuan, Naira/Euro, Naira/USD and Naira/Pound. The next highest estimate is Spearman's ρ rank correlation, while Kendall's τ gave the lowest estimate of correlation. Also, all the estimated correlation measures are positive but weak dependence is reported by Kendall's τ, while Pearson correlation coefficient *r* produced a strong dependence estimate. It should be noted that Kendall's τ estimates are often lower than those of Pearson correlation coefficient *r* (Alexander, 2008).

In terms of dependence of Naira/Yuan against other exchange rates using Pearson correlation coefficient, the highest dependence is obtained against Naira/USD followed by Naira/Pound with Naira/Euro having the least. For the highest dependence using Kendall's τ and Spearman's ρ for Naira/Yuan, Naira/Euro is the highest followed by Naira/Pound, while Naira/USD is the least.

For the estimated Pearson correlation of Naira/Euro against other exchange rates, the highest dependence is obtained against Naira/Pound followed by Naira/USD with Naira/Yuan having the least value. For the highest dependence value using Kendall's τ and Spearman's ρ against the Naira/Euro, Naira/Pound produced the highest dependence followed by Naira/Yuan, while Naira/USD is the least.

Table 3: Parameter estimates for the GJR-GARCHleft(1,1) Conditional Variance

Exchange Rate Pairs		SKEWED t errors	STUDENT t errors	NORMAL errors
Naira/Yuan	ω	0.0193	0.0172	0.0443
	α	0.7904	0.7873	0.3137
	γ	-0.0743	-0.0885	0.8836
	β	0.2466	0.2568	0.2443
	ν	2.6176	2.6643	
	λ	-0.0610		
		LL=463.036	LL=456.569	LL=-793.962
Naira/Euro	ω	0.0159	0.0156	0.0336
	α	0.0530	0.0531	0.4350
	γ	-0.0530	-0.0531	-0.4223
	β	0.9286	0.9288	0.7760
	ν	3.4144	3.4465	
	λ	-0.0425		
		LL= -1.561e+03	LL= -1.563e+03	LL= -2.298e+03
Naira/USD	ω	0.0754	0.0004	0.0022
	α	0.9998	0.3951	0.3356
	γ	-0.0000	0.0835	0.6918
	β	0.0000	0.3839	0.3183
	ν	2.0100	5.1413	
	λ	-0.6824		
		LL= 5.202e+03	LL= 3.776e+03	LL= 2.196e+03
Naira/Pound	ω	0.0684	0.0686	0.0110
	α	0.1657	0.1633	0.2075
	γ	-0.0335	-0.0299	-0.1715
	β	0.6752	0.6735	0.8780
	ν	4.1196	4.1424	
	λ	-0.0353		
		LL= -1.691e+03	LL= -1.692e+03	LL= -1.996e+03

Table 4: Estimated Dependence

Exchange Rate Pairs	Correlations			
	Naira/Yuan	Naira/Euro	Naira/USD	Naira/Pound
Naira/Yuan	Pearson	0.7459(0)	0.9282(0)	0.8147(0)
	Kendall	0.1975 (0)	0.1042(0)	0.1743(0)
	Spearman	0.2839(0)	0.1294(0)	0.2497(0)
Naira/Euro	0.7459(0)	Pearson	0.7570(0)	0.8046(0)
	0.1975(0)	Kendall	0.0483	0.3116(0)
	0.2839(0)	Spearman	0.0614	0.4424(0)
Naira/USD	0.9282(0)	0.7570(0)	Pearson	0.7874(0)
	0.1042(0)	0.0483	Kendall	0.0573(0)
	0.1294(0)	0.0614	Spearman	0.0729(0)
Naira/Pound	0.8147(0)	0.8046(0)	0.7874(0)	Pearson
	0.1743(0)	0.3116(0)	0.0573(0)	Kendall
	0.2497(0)	0.4424(0)	0.0729(0)	Spearman

The Pearson correlation coefficient between Naira/USD and other exchange rates shows that the highest dependence is obtained against Naira/Yuan followed by the Naira/Pound with Naira/Euro giving the least. For the highest dependence value using Kendall's τ and Spearman's ρ for Naira/USD, Naira/Yuan produced the highest value followed by Naira/Pound, while Naira/Euro gave the lowest value.

The Pearson correlation coefficient between Naira/Pound and other exchange rates shows that the highest dependence is obtained against Naira/Yuan followed by the Naira/Euro with Naira/USD producing the least value. For the highest dependence value using Kendall's τ and Spearman's ρ for Naira/Pound, Naira/Euro produced the highest value followed by Naira/Yuan, while Naira/USD gave the lowest value.

Generally, the gap between the lowest and highest estimated dependence measures for an exchange rate against others is least, for Pearson correlation coefficient and is highest, by a large magnitude, for Kendall's τ and Spearman's ρ. Also, the ordering of dependence from highest to lowest or vice versa among the exchange rates is the same for both Kendall's *tau* and Spearman's ρ but different for Pearson correlation coefficient. These findings can have profound impact for risk, portfolio and investment managers.

4.2 Constant Copula models

Table 5 shows the estimated value of parameters for the 8 different models for the copula of the standardized residuals of the indices. The value of the estimated copula parameter is given under the copula family and the next value, the log-likelihood is given below the parameter estimated. The optimal copula (in terms of log-likelihood) is one with lowest likelihood. The top three models for each exchange rate pair are highlighted in bold.

Table 5: Estimated constant copula parameter and log-likelihood values for constant copula

	Normal	Student-t	Clayton	Gumbel	Rotated Gumbel	SJC
Naira/Yuan	0.2989	0.3094,	0.4075	1.2335	1.2425	0.1316,
vs	-96.5069	4.7674	-94.3577	-105.1202	**-118.0229**	0.1748
Naira/Euro		**-140.7810**				**-128.2393**
Naira/Yuan	0.1570	0.2433,	0.1381	1.1000	1.1000	0.0109,
vs	-4.1885	99.9833	-4.2143	0.0237	**-4.9246**	0.0427
Naira/USD		**-5.5194**				**-5.6727**
Naira/Yuan	0.2853	0.2781,	0.3867	1.2180	1.2274	0.1232,
vs	-87.5175	4.0282	-89.1269	-97.9118	**-112.7706**	0.1689
Naira/Pound		**-136.4735**				**-124.4919**
Naira/Euro	0.1436	0.2483,	0.1412	1.1000	1.1000	0.0154,
vs	**-5.5412**	99.8655	-4.2308	-0.4196	-5.2304	0.0471
Naira/USD		**-5.5435**				**-5.9510**
Naira/Euro	0.4829	0.4823,	0.7529	1.4493	1.4692	0.2824,
vs	-273.7073	4.1870	-244.0832	-269.9869	**-293.3602**	0.3461
Naira/Pound		**-316.5257**				**-315.9681**
Naira/USD	0.1446	0.2522,	0.1441	1.1000	1.1000	0.0171,
vs	**-5.7868**	99.9735	-4.3628	-0.5918	-5.4269	0.0492
Naira/Pound		**-5.7584**				**-6.1409**

Table 5 shows that for the Naira/Yuan vs Naira/Euro, Naira/Yuan vs Naira/Pound as well as Naira/Euro vs Naira/Pound exchange rates, the best copula model that captures the dependence is the *t* copula, followed by the SJC copula and then the "Rotated Gumbel" copula.

It is worthy of note that all the copula functions have positive parameters, indicating that Naira exchange rate correlates positively with other exchange rates. For the normal copula, the highest dependence parameter value is obtained from Naira/Euro vs Naira/Pound (0.4829) followed by Naira/Yuan vs Naira/Euro (0.2989). The least dependence is obtained by Naira/Euro vs Naira/USD (0.1436) and Naira/USD vs Naira/Pound (Naira/USD vs Naira/Pound). In terms of Student t copula, the highest dependence parameter is from Naira/Euro vs Naira/Pound (0.4823), while the least is from Naira/Yuan vs Naira/USD (0.2433). Clayton copula reported 0.7529 as its highest value of the dependence parameter for Naira/Euro vs Naira/Pound exchange, while the last dependence value was 0.1381 for Naira/Yuan vs Naira/USD among all the exchange rate pairs. Both Gumbel and "Rotated Gumbel" reported the highest dependence parameters for Naira/Euro vs Naira/Pound and the least values were for the other 3 exchange rate pairs. The SJC copula also chose Naira/Euro vs Naira/Pound as having the highest dependence parameter.

It is important to choose exchange rates with both high as well as low dependence, together, in a portfolio for diversification purposes. For the Naira/Yuan vs Naira/Euro, the worst model is the Clayton copula, followed by Gaussian copula and then Gumbel as the next least ideal model for this exchange rate dependence. The emergence of student t copula as the best performing copula for dependence measurement of exchange rates collaborates the finding of Dias (2004).

The implication of the best models for the Naira/Yuan vs Naira/Euro, Naira/Yuan vs Naira/Pound as well as Naira/Euro vs Naira/Pound exchange rates is that there is asymmetry as well as tail dependence in the dependence structure which Gaussian copula or multivariate normality cannot account for.

For Naira/Yuan vs Naira/USD, the best copula model that captures the dependence is the SJC copula, followed by the Student t copula and then the "Rotated Gumbel" copula. By far the worst model is the Gumbel copula, followed by Gaussian copula and then Clayton as the next least ideal model for this exchange rate dependence. For Naira/Euro vs Naira/USD, the best copula model that captures the dependence is the SJC copula, followed by the Student t copula and then the Normal copula. By far the worst model is the Gumbel copula, followed by Clayton copula and then "Rotated Gumbel" as the least ideal model for this exchange rate dependence.

As far as the Naira/USD vs Naira/Pound exchange rate is concerned, the best copula models for the dependence structure in decreasing order of goodness-of-fit are: SJC, Normal and Student t. The copula models with the worst goodness-of-fit are Gumbel, followed by Clayton and then "Rotated Gumbel" copula models.

From this analysis we can conclude that the Naira exchange rates can be modelled with student t-copula or the identified copula model for the exchange rate, together with univariate t-distributions for the margins (the AR–GJR-GARCH (1,1) models described above with Hansen's skewed *t* distribution of the residuals). Equipped with the copula-based Nigeria exchange rate models, risk measures (like Value-at-Risk or expected shortfall) or functionals of those returns like portfolio expected value can be estimated after simulating from the identified distributions.

4.3 Time-Varying Copula

Figure 3 portrays dependence between selected Naira exchange rate pairs as time-varying similar to volatility. The plots are selected for Gaussian copula, "Rotated Gumbel" Copula and SJC.

Table 6 shows the results for the 8 different time-varying models for the copula of the standardized residuals of the indices. The value of the estimated copula parameter is given under the copula family and the log-likelihood, is given below the parameter estimated. It should also be noted that the optimal copula (in terms of log-likelihood) is one with the lowest likelihood. The top three models for each exchange rate pair is highlighted in bold.

Table 6 shows that for the Naira/Yuan vs Naira/Euro, Naira/Yuan vs Naira/Pound as well as Naira/Euro vs Naira/Pound exchange rates, the best copula model that captures the dependence is the *t* copula, followed by the SJC copula and then the "Rotated Gumbel" copula. This is the same models selected as the best fit for the exchange rate series by constant copula models.

For the Naira/Yuan vs Naira/Euro and Naira/Yuan vs Naira/Pound, the worst model is the Gumbel copula, followed by Clayton copula and then Gaussian as the next least ideal model for this exchange rate dependence. For the Naira/Yuan vs Naira/Euro, Naira/Yuan vs Naira/Pound as well as Naira/Euro vs Naira/Pound, the worst model is the Gumbel copula, followed by Clayton copula and then Gaussian as the next least ideal model for this exchange rate dependence.

For Naira/Yuan vs Naira/USD, the best copula model that captures the dependence is the SJC, followed by "Rotated Gumbel" and then the Gumbel copula. By far the worst model is the Student t copula, followed by Gaussian copula and then Clayton as the next least ideal model for this exchange rate dependence. For Naira/Euro vs Naira/USD, the best copula model that captures the dependence is “Rotated Gumbel” followed by the SJC and then Clayton copula. By far the worst model is the Gumbel, followed by Gaussian and then Student t copula for this exchange rate dependence.

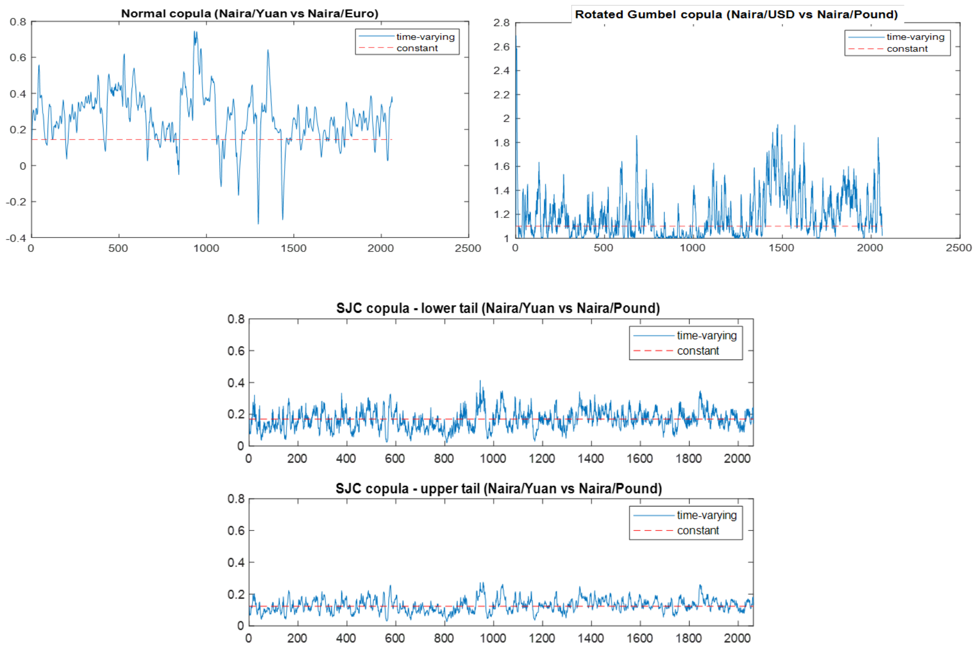

Figure 3: Dependence concept plotted for constant and time-varying copula

As far as the Naira/USD vs Naira/Pound exchange rate is concerned, the best copula models for the dependence structure in decreasing order of goodness-of-fit are: SJC, Rotated Gumbel” and Clayton. The copula models with the worst goodness-of-fit are Gumbel, followed by Gaussian and then Student t copula models.

It is worthy of note that none of the constant or time-varying copula models selected Gaussian copula as the best model for dependence between any of the 4 exchange rate pairs. Recall that for the Naira/Yuan vs Naira/Euro, Naira/Yuan vs Naira/Pound as well as Naira/Euro vs Naira/Pound exchange rates, the best copula model that captures the dependence is the student *t* copula, followed by the SJC copula and then the “Rotated Gumbel” copula.

Table 6: Estimated copula parameters and log-likelihood (LL) values for Time-varying copula

Parameters	Normal (ω, α, β) LL	Student-t $(\omega^1,\alpha^1,\beta^1, \omega^2,\alpha^2,\beta^2)$ LL	Clayton (ω, α, β) LL	Gumbel (ω, α, β) LL	Rotated-Gumbel (ω, α, β) LL	SJC $(\omega^L,\alpha^L,\beta^L, \omega^U,\alpha^U,\beta^U)$ LL
Naira/Yuan vs Naira/Euro	0.0353, 0.1371, 1.8185 -130.2998	0.0321, 0.0873, 1.8706- 1.7503, -0.4790, -0.1828 **-163.9817**	1.3013, 0.0139, -2.6494 -116.3595	0.8946, -1.1103, -0.0268 -94.3427	0.6657, 0.1832, -1.5559 **-138.3575**	-0.4762, -6.0148, 2.0574 4.1035, -23.1177, -2.9650 **-159.3294**
Naira/Yuan vs Naira/USD	1.099, -2.337,- 1.5816 -7.2166	0.0954, 0.3017,- 0.9497, 1.4554, 6.7731, -1.7560 47.2178	1.628,0.1729, -5.0009 -55.1298	1.1757, 0.1974 -4.2037 **-81.8242**	1.0322, 0.2199, -3.5073 **-82.1034**	3.4897, -25.000, 0.1877 3.4981, -25.000, -0.2291 **-94.3445**
Naira/Yuan vs Naira/Pound	1.009, 0.4104, -2.1258 -98.4028	0.7768, 0.2088, -1.1695, -2.3929, -0.2297, -0.2957 **-142.4870**	1.1999, -0.4186 -1.5560 -94.7090	0.8788, -1.1008. -0.0140 -94.7309	1.3838, -0.4863 -1.1387 **-118.0989**	-0.6887, -4.908, 0.2610 0.979, -8.0455, -2.9902 **-130.6753**
Naira/Euro vs Naira/USD	0.3888, -0.5703, 0.6888 -6.3633	1.1507, -0.349, -2.161, 2.8307, -2.4333, 0.1855 -6.4793	1.1389, 0.3502 -3.1244 **-18.3077**	0.8821, -1.1355, 0.2234 -5.4464	0.1411, 0.5508 -1.7144 **-24.2656**	1.6764, -19.4369, 1.3197 0.1264, -10.9466, 3.2968 **-23.6466**
Naira/Euro vs Naira/Pound	1.5829, 0.2406, -1.3169 -280.8270	-0.0305, 0.0491, 2.1921, -1.7668,- 0.1420,- 0.4160 **-327.1384**	1.6844, -0.2467, -2.7244 -267.1224	0.9164,- 1.0937 -0.0125 -103.6168	1.6206,- 0.2491 -2.4267 **-324.4213**	0.1191, -6.3779, 1.4031 1.9447, -8.0488, -2.3966 **-347.4549**
Naira/USD vs Naira/Pound	0.0621,- 0.1473,1.8676 -7.8687	1.0608,- 0.156,- 2.1598 -1.546, 6.7141, 0.0228 -9.4018	1.5593, -0.3685 -3.9408 **-9.4421**	0.8874,- 1.1206, 0.0680 -3.9247	1.3999, -0.0285 -4.1210 **-20.5389**	3.9719, -24.997, -1.9792 4.252, -24.9982, -3.4057 **-27.5364**

For Naira/Yuan vs Naira/USD, the best copula model that captures the dependence is the SJC, followed by "Rotated Gumbel" and then the Gumbel copula. For Naira/Euro vs Naira/USD, the best copula model that captures the dependence is "Rotated Gumbel" followed by the SJC and then Clayton copula. As far as the Naira/USD vs Naira/Pound exchange rate is concerned, the best copula models for the dependence structure in decreasing order of goodness-of-fit are: SJC, Rotated Gumbel" and Clayton.

We also compared the constant copula against the time-varying models for the same exchange rate pair using the estimated log-likelihood as shown in tables 5 and 6. For instance, the constant Gaussian copula produced -96.51 but the time-varying gave -130.30 as the log-likelihood Naira/Yuan vs Naira/Euro. For the Naira/Yuan vs Naira/Euro, all the time-varying copula models outperformed their constant counterparts as ideal dependence models except for Gumbel. From tables 5 and 6, the log-likelihood values show that the time-varying models were better fit than their corresponding constant counterparts for almost all the exchange rate pairs, in some cases by a significant margin. Also, as seen from figure 2, Naira exchange rate dependence is indeed time-varying given the oscillations observed in the plotted dependence between respective Naira exchange rate pairs.

5. Conclusion and Policy Implications

This study analysed the Naira exchange rate co-movements and dependence against four currencies (USD, Euro, Yuan and Pounds) exchange rates from 23 December 2011 to 12 May 2020. The paper also examined asymmetry in the dynamic dependence between the Naira and USD, Euro, Yuan and Pounds exchange rates. We fitted constant and time-varying copulas to the standardized residuals on the four exchange rates under different copula distribution assumptions: Gaussian, student-t, Gumbel, Rotated Gumbel, Clayton and SJC.

The results indicate that the Naira versus the other three currency exchange rates have negative skewness, reflecting asymmetry in the distributions. Additionally, all the exchange rate returns showed evidence of fat tails, indicating non-normality. The study also revealed that Pearson correlation coefficient produced the highest estimate for Naira/Yuan, Naira/Euro, Naira/USD and Naira/Pound indicating strong dependence, while Kendall's tau gave the lowest estimate suggesting weak dependence. The student-t copula was revealed to be the best constant and time-varying copula for modelling dependence among the Naira/Yuan vs Naira/Euro, Naira/Yuan vs Naira/Pound, and Naira/Euro vs Naira/Pound exchange rates. It is important to note that none of the constant or time-varying copula models selected Gaussian copula as the best model for dependence between any of the four exchange rate pairs.

The implication of the best models is that there is asymmetry as well as tail dependence that Gaussian copula or multivariate normality cannot account for. It is therefore important to choose exchange rates with both high as well as low dependence, together, in a portfolio for diversification purposes.

This, therefore, implies that portfolio diversification like the asset allocation of CBN Foreign Reserves as well as risk management decisions should not be based on linear correlation coefficient (Gaussian copula) but should rely on copula models that can capture asymmetry and tail dependence like Student t, SJC and Gumbel copulas.

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