Volatility in the Nigerian Stock Market: Empirical Application of Beta-t-GARCH Variants

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The Generalized Autoregressive Score (GAS), Exponential GAS (EGAS) and Asymmetric Exponential GAS (AEGAS) are new classes of volatility models that simultaneously account for jumps and asymmetry. Using these models, we estimate the dynamic pattern of the Nigeria All Share Index (ASI) from January 3, 2006 to July 22, 2014. Parameter estimates of the models were obtained using the Quasi Maximum Likelihood (QML) approach, and in-sample conditional volatility forecasts from each of the models were evaluated using the minimum loss function approach. Among the classical volatility models, the initial results detected IGARCH-t as the best model for predicting volatility in the ASI. However, in estimating the GAS variants, the Beta-t-EGARCH model proves to predict the volatility in the stock returns better than the IGARCH-t. The estimates could not improve further when the skewed version of the Student-t distribution was considered. We therefore recommend the GAS, EGAS and AEGAS family models in predicting jumps, outliers and asymmetry in financial time series modelling.

Keywords: Asymmetry, Beta-t-GARCH, Generalized Autoregressive Score, Jumps, Nigerian Stock Market.

JEL Classification: C22

1. Introduction

The importance of financial assets as it affects global economy coupled with global financial crisis of 2008 has gingered researchers, regulators and financiers towards studying the dynamic patterns of assets time series. A lot of progress has therefore been made in their understanding of dynamics and distributions of risks (volatility). Volatility, a measure of risks is compounded by the identification of properties/stylized facts in stocks returns, and by development and applications of methodologies for investigating and estimating these time series.

The classical Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model of Bollerslev (1986) and its asymmetric variants are not robust enough to capturing occasional changes in financial series

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known as jumps, thereby tends to underestimate the magnitude of the returns.[4][5] The GARCH model assumes that probability distributions are also limited in this regards, therefore mixture of probability distributions are necessary to predict well, the dynamics of conditional volatility realized from such returns series. The Beta probability distribution, mixed with the Student-t distribution and the resulting mixed-distribution applied to the GARCH model, with little modification to obtain the volatility model that is robust in modelling jumps. The oil and stocks market stress of 1987 and 2008-2009, respectively are very good examples of jumps in volatility series (see Bates, 2000; Pan, 2002). These are occasional large movements with transient impact. For example, Eraker, Johannes and Polson (2003) apply continuous time stochastic volatility models with jumps components in returns and volatility of S&P500 and Nasdaq stocks indices and observe significant evidence of jumps components, both in the volatility and in the returns.

Generalized Autoregressive Score (GAS), the Exponential GAS (EGAS) and the Asymmetric Exponential GAS (AEGAS) are new classes of volatility models that simultaneously account for jumps and asymmetry. These models are the variants of Beta-t-GARCH family recently proposed in the literature by Creal, Koopman and Lucas (2013) and Harvey (2013). To the best of our knowledge, these Beta-t-GARCH models have not been applied to model volatility in the Nigeria Stock Market, but existing literature focus on classical symmetric and asymmetric GARCH models (see Ogum and Nouyrigat 2005, Osazevbaru 2014 and Atoi 2014).

Therefore, this study, seeks to estimate volatility in the Nigerian Stock Market using the recently proposed jumps robust volatility models, which account for jumps and asymmetry inherent in financial returns with the view to comparing the estimated models with the choice of IGARCH-t model for ASI series in Yaya (2013) and recommending the most appropriate model for financial Analysts and portfolio managers in the financial market. These jumps in ASI were experience as a result of influence of news, politics and global crisis on the Nigerian economy.

[4] Salisu (2016) related jumps with structural breaks in oil price series where the author modelled oil price volatility using Beta-Skew-t-EGARCH framework.
[5] Details about the definition of asymmetry in financial series if found in Black (1976).

The structure of the paper is as follows: Section 1 introduces the paper. Section 2 presents the literature on jumps and Beta-t-GARCH models. Section 3 presents the variants of Beta-t-GARCH models and the estimation method. Section 4 presents the data and empirical results, while section 5 renders the summary, conclusion and policy recommendations.

2. Review of Literature

All share index data are good examples of financial time series data, where behaviour of stock could be investigated and future market volatility predicted. The financial models are mostly volatility models, and the GARCH model of Bollerslev (1986); Exponential GARCH (EGARCH) model of Nelson (1991); Glosten, Jaganathan and Runkle (GJR) model of Glosten, Jaganathan and Runkle (1993), Asymmetric Power ARCH (APARCH) model of Ding, Granger and Engle (1993) are all stationary models. The nonstationary volatility model is the Integrated GARCH (IGARCH) model proposed in Engle and Bollerslev (1986). All these models were proposed based on the innovations assumptions of volatility model. These assumptions are those of normality, Student–t and generalized error distributions, as well as the skewed versions. In practical applications, the dynamics/distributions of the volatility series may not follow any of the six probability distribution combinations above. Xekalaki and Degiannakis (2010) believe that the unconditional distribution of the innovations are always thicker than the normal distribution, therefore this calls for mixture of probability distributions that can capture well the tail effect of the volatility series.[6] Financial time series data exhibit leptokurticity, that is, the distribution of their returns is fat-tailed (see Mandelbrot, 1963; Fama, 1965).

A promising GARCH distribution that models both the skewness and the kurtosis is the skewed-Student t- distribution proposed in Fernandez and Steel (1998). This probability distribution was introduced in GARCH modelling in Lambert and Laurent (2000, 2001). As noted in Laurent (2013), the returns from asset prices occasionally exhibits large changes that cannot be attributed to outliers, but jumps in the real sense. These jumps were modelled using Poisson or Bernoulli jump distribution, which when combined with other probability distribution, leads to Poisson or Bernoulli mixtures of GARCH probability distributions for

[6] Financial time series exhibits leptokurticity, that is, the distribution of their returns is fat-tailed (see Mandelbrot, 1963; Fama, 1965).

financial returns. Some studies have assumed fat tails probability distributions, such as the skewed or unskewed Student-t distribution or the generalized error distribution to account for the occurrence of large changes in returns. Meanwhile, these jumps may affect future volatility less than what standard volatility would predict. Other volatility models that are not robust to jumps may substantially inflate the realized measures whenever there are jumps (see Lee and Mykland, 2008). Huang and Tauchen (2005) show the importance of jumps in volatility series and argue that they account for up to 7 percent of Standard & Poors (S&P500) cash index variation.

The arrival of new information could lead to unexpected rapid changes/jumps in the prices of stocks, with general price movement following the Brownian motion, which means the present state is being determined directly by the past state. Jumps in stock prices are also known to follow a probability law, that is, a Poisson process which is a continuous time discrete process (Laurent, 2013). Andersen, Bollerslev and Dobrev (2007) initially propose Brownian Semi-Martingale with Jumps (BSMJ) models for predicting prices of financial assets, but this model is not parametric and cannot predict the market volatility[7].
The specification of GARCH model is based on the assumption that each return observation has the same relative impact on future conditional volatility, regardless of the magnitude of the returns. With an increasing body of evidences, it has been shown that largest return observations may have a relatively smaller effect on future volatility than smaller shocks. Creal, Koopman and Lucas (2011, 2013) and Harvey (2013) propose models to deal with volatility modelling in the presence of jumps in the returns series, and further work on the model has been presented in Creal, Koopman and Lucas (2014) and Blasques, Koopman and Lucas (2014a,b). Their models rely on a non-normal distribution of the innovations and GARCH-type equation for the conditional variance derived from the conditional score of the assumed GARCH probability distribution with respect to the second moment. This class of models is termed Generalized Autoregressive Score (GAS) models, which gives

[7] Non-parametric tests for detecting of jumps in financial time series are well documented in Anderslev and Bollerslev (1997), Andersen, Bollerslev, Diebold and Labys (2003), Andersen, Bollerslev, Christoffersen and Diebold (2006), Andersen, Bollerslev and Diebold (2007), Anderslev, Bollerslev and Dobrev (2007), Andersen, Dobrev and Schaumburg (2008), Barndorff-Nielsen and Shephard (2004a,b), Barndorff-Nielsen and Shephard (2006), Lee and Mykland (2008), Boudt, Croux and Laurent (2008, 2011a, 2011b).

rise to new interesting models such as the dynamic models for location, volatility, and multivariate dependence for fat-tailed densities of Creal, Koopman and Lucas (2011).

The extension to this class of models is presented as the observation-driven mixed measurement dynamic factor models proposed in Andres (2014), Creal, Koopman and Lucas (2014), Harvey and Luati (2014) and Lucas, Schwaab and Zhang (2014). This family encompasses well-known observation driven time varying parameter models including the Autoregressive Conditional Heteroscedasticity (ARCH) model of Engle (1982), the GARCH model, the EGARCH model, the Autoregressive Conditional Duration (ACD) model of Engle and Russell (1998), the Multiplicative Error Model (MEM) of Engle (2002), the Autoregressive Conditional Multinomial (ACM) model of Rydberg and Shephard (2003), and many other related models. Among these models are the Beta-t-EGARCH models which combine the dynamics of time-varying parameter using the scaled score of the conditional density (Blasques, Koopman and Lucas, 2014b). The Beta-skew-t-GARCH model is specified with the conditional probability distribution that is heavily tailed and skewed, and this model is found to perform better than GARCH model with skew Student-t distribution (Harvey, 2014). The exponential and asymmetric versions of these models were also proposed (see Harvey, 2013).

3. Methodology

The classical GARCH model and its asymmetric variants are not robust to jumps inherent in return series, thus, policy decisions arising from their parameter estimates could be misleading.

3.1 The Beta-t-GARCH Variants

We define r_t as a $(t \times 1)$ vector of assets log-returns up to time *t*, that is,

$$r_t = \varepsilon_t = \sigma_t z_t, \tag{1}$$

where z_t follows a particular probability distribution[8], and σ_t is the square root of the conditional variance. The GARCH(1,1) equation of Bollerslev (1986) is given by,

[8] The unconditional distribution of GARCH model still gives thicker tails than the normal distribution, therefore other classical distributions like Student-t (Bollerslev, 1987), GED

$$\sigma_t^2 = w + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (2)$$

where w, α_1 and β_1 are parameters, and these, conditioned as $w > 0$, $\alpha_1 \geq 0$, and $\beta_1 \geq 0$ to ensure positive definiteness and covariance stationarity of the conditional variance series. Re-writing GARCH (1,1) in (2) gives,

$$\sigma_t^2 = w + \alpha_1 z_{t-1}^2 \sigma_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$
$$\sigma_t^2 = w + \alpha_1 \left(z_{t-1}^2 - 1\right)\sigma_{t-1}^2 + \left(\alpha_1 + \beta_1\right)\sigma_{t-1}^2$$
$$\sigma_t^2 = w + \alpha_1 u_{t-1} \sigma_{t-1}^2 + \varphi_1 \sigma_{t-1}^2 \quad (3)$$

where $\varphi_1 = \alpha_1 + \beta_1$ and $u_t = z_{t-1}^2 - 1$ for Normal distribution is proportional to the score of the conditional distribution of ε_t with respect to σ_{t-1}^2. The model (3) is Beta-N-GARCH model[9], since $\left(u_t + 1\right)/\left(v + 1\right)$ has a Beta distribution, and the innovations u_t are given as, $u_t = \frac{\left(v+1\right) z_t^2}{v - 2 + z_t^2} - 1$ for Student-t distribution, $z_t \approx T\left(0,1,v\right)$ (4)

and $u_t = \frac{\left(v+1\right) z_t z_t^*}{\left(v-2\right) g_t \xi^{I_t}} - 1$ for Skewed Student-t distribution, $z_t \approx skT\left(0,1,\xi,v\right)$ where[10]

$$z_t^* = s z_t + m,\ I_t = sgn\left(z_t^*\right) = I\left(z_t^* \geq 0\right) - I\left(z_t^* < 0\right),\ g_t = 1 + \frac{z_t^{*2}}{\left(v-2\right)\xi^{2I_t}},$$

(Nelson, 1991) as well as the skewed versions (see Hansen, 1994; Lambert and Laurent, 2000, 2001; Theodossiou, 2002)) are often applied. These skewed versions were proposed to account for both fat left and right tails of the GARCH innovations.

[9] By setting $\psi_t = \sigma_t^2$, and $\kappa_t = S_t \nabla_t$, where ∇_t is the score with respect to the parameter ψ_t that is $\nabla_t = \partial \log f\left(y_t \middle| \psi_t, \Psi_{t-1}, Y_{t-1}, X_t; \theta\right) / \partial \psi_t$ and S_t is a time domain dependent scaling matrix set as $S_t = 2$. Equating $\alpha_1 = A_1$ and $\alpha_1 + \beta_1 = B$, the Beta-GARCH model is re-specified in a more compact form as $\psi_t = w + A_1 \kappa_{t-1} + B_1 \psi_{t-1}$.

[10] Note, $I_t = sgn\left(z_t^*\right) = I\left(z_t^* \geq 0\right) - I\left(z_t^* < 0\right)$ is an indicator measure asymmetry.

$$m = \frac{\Gamma\left(\frac{v-1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\Gamma\left(\frac{v}{2}\right)}\left(\xi - \frac{1}{\xi}\right) \text{ and } s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2} \,. \qquad (5)$$

Now, combining (3) with (4) gives the Beta-t-GARCH model and combining (3) with (5) gives the Beta-Skew-t-GARCH model. The Student-t distribution is symmetric, just like the Normal distribution and it is expected to capture the tail effect better than the normal distribution. The skewed Student-t distribution is asymmetric, expected to perform better than the symmetric distribution in predicting the tail effect volatility model.

Harvey (2013) also considered the Exponential GARCH (EGARCH) and Asymmetric Exponential GARCH (AEGARCH) types of the Beta-GARCH models, each with the two distributional assumptions[11] applied. The Beta-EGARCH model, specified without the leverage effect[12], is given as:

$$\log \sigma_t^2 = w + \alpha_1 u_{t-1} + \varphi_1 \log \sigma_{t-1}^2 \qquad (6)$$

Introducing the leverage effect, we have the Beta-AEGARCH model,

$$\log \sigma_t^2 = w + \alpha_1 u_{t-1} + \gamma_1 l_{t-1} + \varphi_1 \log \sigma_{t-1}^2 \qquad (7)$$

where $l_{t-1} = sgn(-z_t)(u_t + 1)$ when Student-t distribution is considered and $l_{t-1} = sgn(-z_t^*)(u_t + 1)$ for the skewed Student-t distribution.[13][14]

Now, combining the Student t distribution in (4) with (6) leads to the Beta-t-EGARCH model, and similarly for skewed Student t distribution.

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[11] Following Harvey (2013), It is very straightforward to consider other GARCH distributional assumptions such as the Normal and Generalized Error distributions once these are sure to capture the tail effect of he innovations very well.

[12] The Beta-EGARCH specification has no asymmetric parameter, unlike the classical EGARCH model of Nelson (1991).

[13] Note, from the definitions of both Student-t and skewed Student-t distributions defined above, $l_{t-1} = sgn(-z_t)(u_t + 1)$ is obtained in a similar manner using the function "*sgn*" (See Laurent, 2013; Harvey, 2013; Salisu, 2016).

[14] Note, $E(l_t) = \frac{1-\xi^2}{1-\xi^2}$ the Student-t distribution and $E(l_t) = 0$ for the Skewed Student-t distribution.

Again, combining the same Student t distribution with (7) leads to Beta-t-AEGARCH model, and we obtain similarly model when skewed Student t distribution is applied. The specifications of GAS models in (6) and (7) closely resemble that of the classical EGARCH(1,1) model of Nelson (1991) with model specification given as,

$$\log\sigma_t^2 = w(1-\beta_1) + (1+\alpha_1 L)\left[\theta_1\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right) + \theta_2\left(\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - E\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right)\right|\right)\right] + \beta_1\log\sigma_{t-1}^2 \quad (8)$$

where L is the lag operator, θ_1 and θ_2 are the asymmetry parameters which control the sign and magnitude respectively. The value of $E(\varepsilon_t/\sigma_t)$ depends on the assumption made on the unconditional density of $z_t = \varepsilon_t/\sigma_t$. For the Skewed Student-t distribution,

$$E\left(\left|\varepsilon_t/\sigma_t\right|\right) = \left(\frac{4\xi^2}{\xi + 1/\xi}\right)\frac{\Gamma\left(\frac{1+v}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\,\Gamma(v/2)} \quad (9)$$

where $\xi = 1$ for the Student-t distribution.

2.1 Maximum Likelihood Estimation and Inference

The procedure for Maximum Likelihood Estimation (MLE) of GAS family models was presented in Blasques, Koopman and Lucas (2014a). The strong consistency and asymptotic normality of the MLE are also presented. The log-likelihood functions, $L(.)$ for the Beta-t-EGARCH model from the Student-t and Skewed Student-t distributions are optimized with respect to the unknown dynamic parameters, contained in the vector Ψ, and *v*; in the first order model, $\Psi' = (w, \alpha_1, \gamma_1, \varphi_1,)$. The likelihood function is known in closed form by means of prediction error decomposition and this facilitates the parameter estimation via the MLE approach.[15] This method uses the Quasi-Newton method, proposed independently in Broyden (1970), Fletcher (Fletcher), Goldfarb (1970) and Shanno (1970), and named after the authors as BFGS algorithm, this was implemented in GARCH7 program by Laurent and Peters (2006) and Laurent (2013).

Straumann and Mikosch (2006) present the asymptotic theory for GARCH models with the complex mathematics. As noted, they were unable to obtain Quasi ML estimates for EGARCH model. Therefore,

[15] See Harvey (2013), Blasques, Koopman and Lucas (2014a, 2014b) for theoretical properties of the MLE for GAS family model.

the use of numerical expressions for simplifying the likelihood function is often applied (see Francq and Zakoian, 2009; 2010).

2.2 Forecasts Comparison

The major essence of modelling in Time Series Analysis is to obtain a representative model for our data, therefore, we subject the estimated conditional volatility series to forecasts performance tests. We apply the Loss Functions approach. Most empirical financial time series papers do not obtain the out-of-sample forecasts for GARCH models or even forecast other nonlinear financial time series, since this is not easy to obtain (Xekalaki and Degiannakis, 2010), we therefore consider only the in-sample-forecast performances of the volatility models.

The usual method of comparing the forecast generated from models by the Mean Square Forecasts Error (MSFE) and Mean Absolute Forecast Error (MAFE) are common in literature. More recent approach is the application of Loss Functions, though these give equivalent results to the naive methods of evaluating forecasts. The Squared Error (SE) and Absolute Error (AE) loss functions are proposed in Brooks and Persand (2003). The Heteroscedasticity-Adjusted Squared Error (HASE) and Heteroscedasticity-Adjusted Absolute Error (HAAE) loss functions are applied in Andersen, Bollerslev and Lange (1999). The Logarithmic Error (LE) loss function is applied in Saez (1997) and the Gaussian Likelihood (GL) loss function is given in Bollerslev, Engle and Nelson (1994). Taking the in-sample conditional volatility series over some τ-day period, then, the in-sample conditional forecasts variance is given as $\hat{\sigma}_{t+1}^{2(\tau)}$ for a period of τ days, from $t+1$ to $t+\tau$ depending on the size of the time series. Since the actual variance for a period of τ business days from $t+1$ to $t+\tau$ is not observed, we therefore apply a proxy measurement of using the squared returns r_t^2 for measuring the actual daily volatility in the log returns from t+1 to t+τ days. The in-sample mean loss functions are then given as,

$$\overline{SE} = 1/\tau \sum_{i=1}^{\tau} \left(\hat{\sigma}_{t+1}^{2(\tau)} - r_{t+i}^2 \right)^2; \tag{10}$$

$$\overline{AE} = 1/\tau \sum_{i=1}^{\tau} \left| \hat{\sigma}_{t+1}^{2(\tau)} - r_{t+i}^2 \right|; \tag{11}$$

$$\overline{HASE} = 1/\tau \sum_{i=1}^{\tau} \left(1 - \left(r_{t+i}^2 / \hat{\sigma}_{t+1}^{2(\tau)} \right) \right)^2; \tag{12}$$

$$\overline{HAAE} = 1/\tau \sum_{i=1}^{\tau} \left| 1 - \left(r_{t+i}^{2} / \hat{\sigma}_{t+1}^{2(\tau)} \right) \right|; \tag{13}$$

$$\overline{LE} = 1/\tau \sum_{i=1}^{\tau} \left[\log \left(r_{t+i}^{2} / \hat{\sigma}_{t+1}^{2(\tau)} \right) \right]^{2} \text{ and} \tag{14}$$

$$\overline{GL} = 1/\tau \sum_{i=1}^{\tau} \left[\log \left(\hat{\sigma}_{t+1}^{2(\tau)} \right) + \left(r_{t+i}^{2} / \hat{\sigma}_{t+1}^{2(\tau)} \right) \right] \tag{15}$$

The model with the smallest loss function actually gives the best forecasts.

3. The Data and Empirical Results

The data used in this paper are the daily All Share Indices (ASI) of Nigerian Stock Exchange from 3 January 2006 to 22 July 2014 covering 2085 data points of business days, excluding weekends and public holidays. The time plot of the ASI is displayed below.

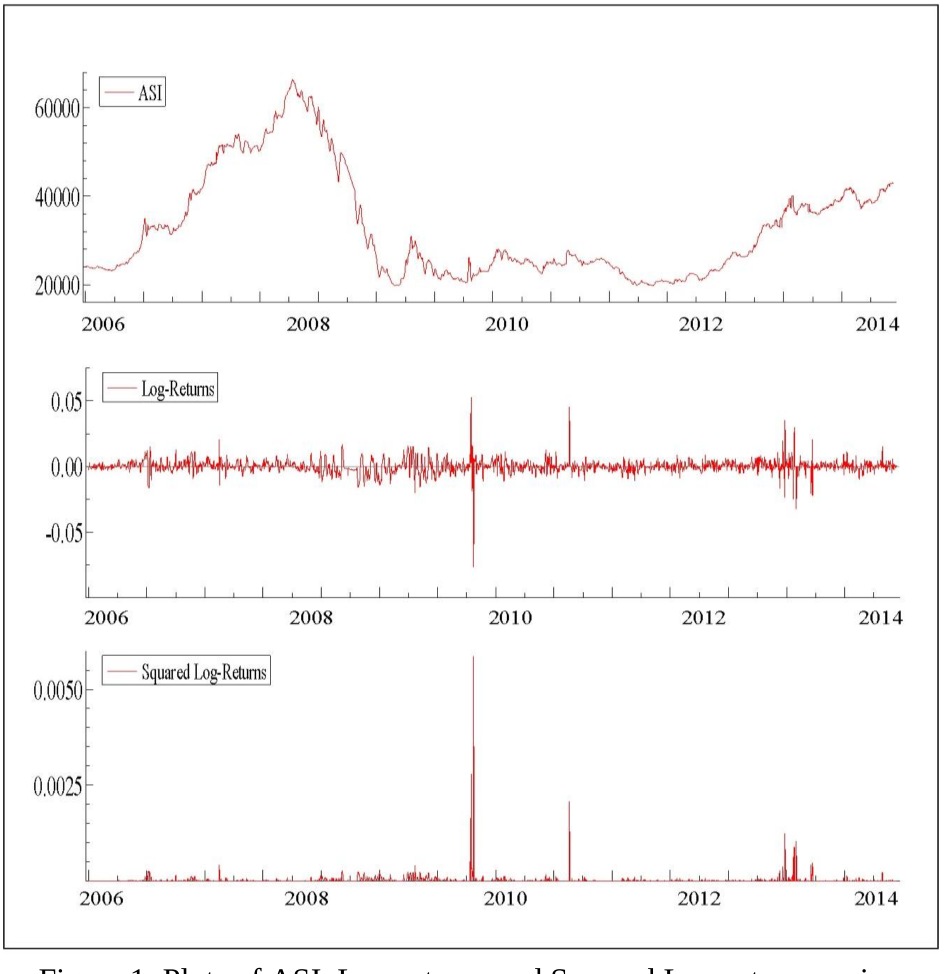

Figure 1: Plots of ASI, Log-returns and Squared Log-returns series

We observe some sharp jumps in the time plot in Figure 1, and as a result, an asymmetric volatility model would likely be the best model for modelling the series (see Salisu, 2012; Yaya, 2013, Yaya and Gil-Alana, 2014). The plots, the log returns of ASI and squared returns of ASI series are given in Figure 1 below. In the upper panel, where we have the stock prices of the (ASI), we observe series of jumps. This, coupled with the asymmetric movement makes modelling volatility in the ASI log-returns complex. Relating the jumps to the log-returns in the middle panel, these jumps correspond to the points with significant spikes. The longest spike around the middle of the plot is as a result of very sharp rise in the prices, which market reacts to occasionally. The squared log-returns are plotted in the lower panel.

We first approached the estimation[16] of the volatility models by estimating the four stationary models and the nonstationary IGARCH models as applied in Yaya (2013), even though we considered different sampled data[17]. We then found the optimal model, out of the five models to be IGARCH, as suggested by the estimates of maximum log-likelihood, minimum Akaike (AIC) and Swartz Bayesian (SBIC) information criteria as given in Table 1. This is in agreement with what is obtained in Yaya (2013), though with different sampled data. The results here are necessary in order to show the superiority of the new class of models over the classical models.

Table 1: Estimates of AIC and SBIC for Classical GARCH Variants

Distribution	*Information Criteria*	*GARCH*	*EGARCH*	*GJR*	*APARCH*	*IGARCH*
Student-t	AIC	-8.2849	-8.2237	-8.284	-8.285	**-8.2858**
	SBIC	-8.2741	-8.2074	-8.2705	-8.2687	**-8.2777**
Skewed Student-t	AIC	-8.2847	-8.2411	-8.2841	-8.2852	**-8.2857**
	SBIC	-8.2712	-8.2221	-8.2678	-8.2663	**-8.2729**

We proceed to estimating the GAS models. Due to the fact that GAS family models were derived from EGARCH specification, we present the results for EGARCH model here and compare the estimates with the

[16] The program used for the estimation is OxGARCH7 software developed by Laurent and Peter (2006) and Laurent (2013).

[17] Yaya (2013) considered ASI time series from January 2007 to December 2011 covering 1231 data points, while the same sample was considered in Yaya and Gil-Alana (2014). The data considered were updated to 2014.

other models, each case, under the GARCH assumed probability distributional assumption. The case of Student-t distribution is given in Table 2, while that of Skewed Student-t distribution is given in Table 3. In Table 2, the full results for IGARCH-t model and EGARCH-t models are presented here. There was algorithm convergence problem, while computing the parameter estimates for EGARCH-t model, therefore only the model evaluation criteria were reported. For the IGARCH-t model, the estimate of excess kurtosis is 55.908. This value is the lowest among kurtosis values of other competing models computed in Table 2. Thus, indicating that the Student-t distribution with the IGARCH specification may not capture well the dynamics of the tail behaviour of ASI log-returns. The three GAS models, the Beta-t-GARCH, Beta-t-EGARCH and Beta-t-AEGARCH gave AIC and SBIC values of (-8.2851, -8.2743), (-8.2871, -8.2762) and (-8.2861, -8.2726), respectively, with that of Beta-t-EGARCH model being the least, making the model the best among the five competing models in Table 2. We can also see that the asymmetric parameter γ_1 is not significant at 5% level. The excess kurtosis for this optimal model is 78.848 and that of Beta-t-AEGARCH model is 78.502. That of Beta-t-GARCH model is 58.957 and this value is very close to that of IGARCH model. Thus, the EGARCH dynamics in the models actually influence the tail distribution.

Table 2: Estimated Volatility models with Student-t distribution assumption

Pars.	**IGARCH (1,1)-t**	**EGARCH (1,1)-t**	**Beta-t-GARCH (1,1)**	**Beta-t-EGARCH (1,1)**	**Beta-t-AEGARCH (1,1)**
w	2.3053***	-0.0098 nc	2.0946***	-1.3E06***	-1.3E06***
α_1	0.4758***	-0.4024 nc	0.4052***	0.3726***	0.3723***
β_1	0.5247***	0.9194 nc	---	---	---
θ_1	---	-12.0015 nc	---	---	---
θ_2	---	100.4108 nc	---	---	---
φ_1	---	---	0.9221***	0.8777***	0.8780***
γ_1	---	----	---	---	-0.0031
Student-t df	4.2547***	2.0000 nc	3.8531***	4.0149***	4.0173***
Model Eval.					
Log-lik.	8636.84	8575.08	8637.1	8639.11	8639.13
AIC	-8.2858	-8.2237	-8.2851	**-8.2871**	-8.2861
SBIC	-8.2777	-8.2074	-8.2743	**-8.2762**	-8.2726
Skewness	2.8249***	3.3708***	2.4526***	**3.4102*** **	3.3964***
Ex. Kurtosis	55.908***	70.355***	58.957***	**78.848*** **	78.502***
Jarque-Bera	274190***	433750***	303920***	**543880*** **	539120***
ARCH(1) test	0.0778	0.0562	1.8503	0.1079	0.0997
ARCH(5) test	0.0567	0.0299	1.6279	0.1556	0.1566
ARCH(10) test	0.0878	0.1027	1.1292	0.1619	0.1611

nc means No Convergence ***significant at 5%.

Considering the case of Skewed Student-t distribution for the models as presented in Table 3, we obtained results for the five models, IGARCH, EGARCH and the three GAS variants. The IGARCH-skew-t model gave AIC and SBIC values of -8.2857 and -8.2729, respectively, with excess kurtosis of 56.501. The EGARCH-skew-t model converged at this time and the two asymmetric parameters are significant. The kurtosis estimate improved to 61.486 due to the introduction of the EGARCH model, though IGARCH-skew-t model would perform better than the EGARCH-skew-t model in terms of fitness. Of the three GAS variants under skewed Student-t distribution, Beta-skew-t-EGARCH model is the optimal, having the least AIC and SBIC of -8.2867 and -8.2731, respectively. The excess kurtosis realized from the model estimates is 77.966.

Now, between Beta-t-EGARCH and Beta-skew-t-EGARCH models, the better model based on minimum information criteria is the Beta-t-EGARCH model.

Table 3: Estimated Volatility models with skewed Student-t distribution assumption

Pars.	IGARCH (1,1)-skew-t	EGARCH (1,1)-skew-t	Beta-skew-t-GARCH (1,1)	Beta-skew-t-EGARCH (1,1)	Beta-skew-t-AEGARCH (1,1)
w	2.257***	-0.1065	2.0536***	-1.3E06***	-1.3E06***
α_1	0.4723***	0.8149***	0.4029***	0.3708***	0.3711***
β_1	0.5277***	0.8148***	---	---	---
θ_1	---	-0.023987***	---	---	---
θ_2	---	0.6186***	---	---	---
φ_1	---	---	0.9232***	0.8792***	0.8789***
γ_1	---	---	---	---	0.0043
Student-t df	4.2716***	4.1109***	3.8617***	4.0219***	4.0189***
Asymmetry	0.0329	0.0378	0.02449	0.0263	0.0286
Tail	4.2716***	4.1109***	3.8617***	4.0219***	4.0189***
Model Eval.					
Log-lik.	8637.68	8594.22	8637.65	8639.72	8639.75
AIC	-8.2857	-8.2411	-8.2847	**-8.2867**	-8.2857
SBIC	-8.2729	-8.2221	-8.2712	**-8.2731**	-8.2695
Skewness	2.8515***	2.4651***	2.4544***	**3.3767*** **	3.3930***
Ex. Kurtosis	56.501***	61.486***	58.714***	**77.966*** **	78.370***
Jarque-Bera	280020***	330380***	301440***	**531790*** **	5.3732***
ARCH(1) test	0.0765	0.0662	1.7995	0.0961	0.1066
ARCH(5) test	0.0561	0.0442	1.5912	0.1652	0.1641
ARCH(10) test	0.0848	0.351	1.1026	0.1658	0.167

***significant at 5%.

We further probe these results by carrying out forecasts evaluation test based on the minimum loss functions on IGARCH, EGARCH and the three GAS variants, for both Student-t distribution and its skewed versions as presented Table 4. The three GAS variants outperformed the IGARCH and EGARCH models in terms of forecasting ability, since the estimates of their loss functions are smaller than that of IGARCH and EGARCH models. Though there were convergence problems, during the estimation of EGARCH model under the Student-t distributional assumption and this was reflected in the very high values of loss functions obtained for the model. Among the three GAS variants, there is no dominating model in terms of forecasts performance.

Table 4: Forecasts Evaluation of models based on Loss Functions Approach

Student-t distribution					
Loss Functions	IGARCH (1,1)-t	EGARCH (1,1)-t	Beta-t-GARCH (1,1)	Beta-t-EGARCH (1,1)	Beta-t-AEGARCH (1,1)
SE	9.09E-08	13.999	8.07E-08	8.02E-08	8.02E-08
AE	6.64E-05	1.0393	5.86E-05	5.70E-05	5.69E-05
HASE	717.5451	0.9997	652.0338	650.2601	651.1461
HAAE	4.5604	0.9998	4.2627	4.5839	4.588
LE	3.577607	29.1704	3.532	3.5598	3.5595
GL	-0.1661	-0.2749	-0.4691	-0.1474	-0.1431
skewed Student-t distribution					
Loss Functions	IGARCH(1,1)-skew-t	EGARCH (1,1)- skew-t	Beta-skew-t-GARCH(1,1)	Beta-skew-t-EGARCH (1,1)	Beta-skew-t-AEGARCH(1,1)
SE	8.24E-08	9.64E-08	7.97E-08	7.97E-08	7.97E-08
AE	5.82E-05	6.26E-05	5.09E-05	5.14E-05	5.14E-05
HASE	943.0013	711.8856	713.0776	618.31	618.1313
HAAE	4.75666	4.2065	4.3461	4.2929	4.2998
LE	3.552	3.5885	3.4377	3.4839	3.4838
GL	0.0565	-0.478	-0.3604	-0.4034	-0.39688

5. Summary, Conclusion and Policy Implications

We have considered modelling the returns of ASI on NSE using variants of Beta-t-EGARCH model. This model, which is asymmetric volatility model was developed from the classical EGARCH model. Its special application of capturing asymmetric jumps in volatility series makes it appealing and preferred to other asymmetric models proposed earlier in literature. As a new model proposed in Harvey (2013), with other GAS family models proposed in Creal, Koopman and Lucas (2011, 2013), the model and its variants are yet to get wider applications, though the asymptotic theory of the models, properties and MLE approach are given in Blasques, Koopman, and Lucas (2014a, 2014b).

In this paper, the ASI time series, as plotted on the time plots indicate different bull and bear market phases, coupled with the global financial crisis between 2008 and 2009. Occasional jumps in prices were observed in the sampled data (January 2006-July 2014). Following Yaya (2013), the IGARCH-t model was picked as the best performing model in predicting the ASI returns, and using ASI with different sample size, the same conclusion was obtained when the classical volatility models (GARCH, EGARCH, GJR, APARCH and IGARCH) were considered. Parameter estimation with minimum information criteria selected Beta-t-EGARCH and Beta-skew-t-EGARCH models under each distributional assumption, and the better model is the Beta-t-EGARCH model. Forecast evaluation test on the five models (IGARCH, EGARCH and the three GAS variants) actually indicated the superiority of GAS variants to IGARCH model, with the EGARCH model as the least performing model in terms of forecasts. Among the three GAS variants, the loss function approach could not distinguish the best performing model.

The dynamics of ASI time series is complex and robust volatility models for jumps are proposed to capture accurately the realized conditional volatility. This paper has shown that the IGARCH-t model of Yaya (2013) may not serve the financiers well in predicting the volatility in the Nigerian stocks market, which led to the proposition of a more robust model. Studies on volatility modelling are expected to be conducted on high frequency data, whereas some other authors erroneously apply monthly data in investigating volatility in stocks, the results obtained therefore underestimate stocks market volatility. Accounting for jumps in investment models is of paramount importance due to its significant implications on assets pricing and portfolio decisions of investors and market players. The combination of decisions of individual market players as well as investors in the economy determines the aggregate market trend. Therefore, as markets are driven by information, it implies that wrong decision taken in respect of inappropriate analytical model (in this case, incorrect capturing of jumps in stock market volatility models) could misinform the market players. Given the interplay of capital and money markets, such misinformation could generate intense market volatility that is capable of distorting the price and financial system stability objective of the monetary authorities.

This paper has established the asymmetric nature of the Nigerian capital market using ASI log-returns as proxy variable to Nigerian stocks. We therefore recommend the GAS, EGAS and AEGAS family model in predicting jumps, outliers and asymmetry in financial time series. These aberrant observations, when left uncaptured in the predicting model could lead to mis-specified model, thereby leading to faulty predictions. This new model can also be applied in predicting the price movements of individual shares/stocks or industry/sub-sectorial indices of the Nigerian stock market, but with the assumptions of data availability. This present work could be extended in many ways: first, by applying it on the price index of each portfolio of NSE; and also on the share prices of individuals stocks.

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