CENTRAL BANK OF NIGERIA

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Non-Technical Summary

Today, the objective of the CBN like any other central banks is the attainment of monetary and price stability usually based on predetermined targets. According to Arestis (2007), the management of inflation has shifted from the attempts to stabilise it around targets to efforts at anchoring inflation expectations. Understanding of economic outlook provides vital information on the evolution of future inflation and helps central banks in their price stability mandates. In the literature, several methods such as the static component, dynamic component and/or both have been applied to forecast economic activity. A number of these techniques are fraught with some challenges: first, they could lead to large forecasting errors; second, such forecasts could mislead economic agents who act based on decision made by monetary and fiscal authorities.This approach allows for the modelling of unobserved components in the time series with the advantage of improving the quality of forecasts germane to central banks' capacity in implementing forward-looking monetary policy.

Thus, this study employed a dynamic factor model to forecast gross domestic product (GDP) growth rate for Nigeria. The justification for this approachis anchored on its ability to extract the underliningfactors required in generating robust and reliable forecast of GDP. The common factor for Nigeria's GDP was derived based on six macroeconomic variables, including GDP growth rate, growth in money supply, credit to the core private sector, government revenue, government expenditure and crude oil production. The forecast period covered 2014Q2 to 2015Q4.

The forecast showed a real GDP growth rate which generally remained above 6.50 per cent. From an actual growthrate of 6.54 per cent in 2014:Q2, output growth was forecasted to rise to 6.93 per cent in 2014:Q3. Nonetheless, a modest slowdown was predicted towards the *end of 2014 and throughout 2015. GDP growth rates for 2015:Q2 and 2015:Q4 were forecasted as 6.83 and 6.75 per cent, respectively. The study also identifiedcomposite leading indicatorsforpeaks and troughs in the real GDP growth. Further analysis suggesteda persistence ofgrowth contraction which lasted about 7.4 months, while the higher growth regime continued for approximately 4.6 months.Thus, in a dynamic economy like Nigeria, the need to regularlyfine-tune this framework becomes imperative.*

Chapter One

1.0 Introduction

The need for a more consistent and accurate GDP forecast for the conduct of forward-looking monetary policy is quite fundamental. This is because the availability of real-time data is crucial to determine the initial conditions of economic activity on latent variables such as the output gap to make policy decisions. Central banks use available monetary policy instruments to influence the volume and direction of monetary aggregates, consistent with predetermined output and price targets. In both developed and developing countries, a traditional central bank reaction function is characterised by price and output development. Thus, taking decisions about the monetary policy rate without an estimate of the output gap is tantamount to 'flying blind' and making unacceptably, large errors and revisions that are uncertain

Several benefits can be derived from generating accurate now-cast of GDP. First, without such reliable forecast, misleading decisions could be taken with far-reaching consequences on the set objectives and forward mis-guidance of economic agents. Second, since output stabilisation is one of the key objectives of the policy maker, accurate output forecast can help to reveal the sacrifice ratio relative to other objectives, such as inflation and exchange rate stability. Third, where structural models are also used for simulation, GDP forecast can feed into these models, thus, enriching mid-term forecast.

Although statutory agencies of government release values of key macroeconomic variables such as output,they come with some lag given the enormity of the data-generating process, particularly the labourious and time-consuming as well as costly field surveys. The lag in the release of actual numbers can be a challenge to the policy makers. For Nigeria, it is even more critical given that the GDP numbers comes with a two-quarter lag. Also, over the last few decades, the economy has witnessed sharp swings during the 1982, 1994 and 2008 global economic crises periods. The pervasiveness of each of these domestic and global phenomena has implications on the ability to measure effectively potential and actual national output.Hence, a forecasting framework is needed to effectively address the challenge of understanding the current state of economic activity for policy decisions.

Since there are several variables with mix frequencies (monthly, quarterly, annually), such as money supply, inflation, stock market capitalisation, whichhave strong correlation with output growth that are readily available on a regular basis.It becomes possible to generate GDP forecast by leveraging on the inherent relationship among these variables. Schulz (2007) has shown that financial and survey data are useful in the forecast of economic activity. In the literature, several methods are used to summarise these large data set for the purposes of forecasting such as the static component, dynamic component and/or both have been applied to forecast economic activity (Allen, 1997; Anderson and Gascon, 2009; and Schulz, 2008). A number of these techniques are fraught with some challenges (Stock and Watson, 2002; Forni, et al., 2003; Schulz, 2008): first, they could lead to large forecasting errors; second, such forecasts could mislead economic agents who act based on decision made by monetary and fiscal authorities.

Thus, this study employed a dynamic factor model (DFM) to forecast gross domestic product (GDP) growth rate for Nigeria to aid monetary policy decisions. This approach allows for the modelling of unobserved components in the time series with the advantage of improving the quality of forecasts germane to central banks' capacity in implementing forward-looking monetary policy. Particularly, this approach is anchored on its ability to extract underlining factors among a set of macroeconomic variables that contain information about the latent state of the economy that can be used in generating robust and reliable GDP forecast. The DFM provide help to addressing the paucity of real-time data. It also permits the utilisation of a mixture of data frequencies in the determination of the current state of the economy. More so, if disturbances suggest absence of normality of the random variables, it gives room for a re-specification of the errors in an autoregressive scheme.

Following this introduction, chapter 2 focuses on the literature review while chapter 3 espouses the stylized facts about business and growth cycles. Chapter 4 covers the methodological issues as the empirical results are considered in chapter 5 while chapter 6 concludes the study.

Chapter Two
2.0 Brief Review of Relevant Related Literature

The extant literature has shown that without good forecast, the conduct of forward-looking monetary policy would be complicated and misleading. Highlighting the importance of forecast in forward-looking monetary policy, Kugler et al., (2004) examined the effect of measurement errors in GDP on inflation and growth volatility in Switzerland. The paper noted that with measurement errors, monetary policy reacted very strongly to noisy data if the weight on output growth targeting becomes too big, as measurement errors have a strong impact on the growth forecast but not on the inflation forecast.

This finding was consistent with earlier studies such as (Orphanides, 2000, 2001) in the US; and Ehrmann and Smets (2003) in the euro area. In the US an examination of real time data for output gap for measurement errors showed that observed measurement errors in the output gap, the neglect of which, they claimed, might result in aggressive policy posture. Thus, errors associated with forecasts in the output gap can be a problem for forward-looking monetary policy. For the euro area, welfare losses in the form of high output variability to measurement problems were traced.

Clarida et al., (2000) argued that if monetary policy would rein in future inflationary pressures, the role of accurate forecasts becomes very germane. Consequently, the paper evaluated forecasts from three central banks of England, Poland and Swiss in forward-looking Taylor rules to ascertain the forward looking nature of their monetary policy decisions. The findings confirmed that the sampled central banks were forward looking. The intuition of this result reflects the necessity of generating key forecasts in the objective function of the central bank for a forward-looking monetary policy.

In terms of implementing forecast, the recognition of GDP series as a veritable input for monetary policy formulation and implementation has led to the adoption of several approaches[1] (structural and time series) in economic literature to forecast the series (see Fenz, Schneider and Spitzer, 2004; Schumacher, 2005; Schulz, 2007; Benkovskis, 2008; Branimir and Magdalena, 2010).The structural models have transited from the Cowles Commission type of models to the more recent dynamic stochastic general equilibrium (DSGE) models. The Cowles Commission models were largely criticised for being ad hoc, policy variant (Lucas critique) and the lack of micro-foundations (see Mankiw, 1991, 2006; Woodford, 1999; and Goodfriend, 2007).

Earlier time series models were the ARIMA models based on Box-Jenkins (1976) approach. In the recent times, the ARIMA models are usually found in studies as benchmark models against which other models are evaluated. A major limitation of the ARIMA model is that it can only be used to predict itself.

This approach was extended in a multivariate application in the vector autoregressions (VARs) following Sims (1980). Studies by Stock and Watson (2001) have shown that VARs have been successful in forecasting. In the class of time series models was the

[1] Broadly speaking, models for economic forecasting can be classified into two groups - time series models and structural models. Time series models are mainly statistical, based on historical developments, traditionally with just a fewvariables and very little, if any, economic content. In structural models, on the other hand, economictheory is used to specify the relationships between the variables, which can be done either by estimationor by calibration.

extension of the VARs by applying larger scale Bayesian VARs as shown the works of Litterman (1986), Sims (1993) and Sims and Zha (1986).BVARs allows for the inclusion of limited variables and with their lags. Again, Bayesian VARs impose some restrictions on the model coefficients, thus, helping to reduce the dimensionality problem of VARs, resulting in more accurate forecasts. A major drawback of the VAR and BVAR among other time series model used for forecasting is it inability to include large number of variables.

In the second class of time series models are the Unobserved Components (Harvey, 2006). This class warehouses the large averaging and empirical Bayes methods; and the factor models in the spirit of Stock and Watson (2006). Thus, in the recent literature, the factor models, which allow the inclusion of large set of latent variables was developed to deal with such challenges (Schumaber, 2005). Factor models such as the static component Stock and Watson (2002) and the dynamic principal component models in the time domain as in Doz et al., (2006 and 2007) and dynamic principal components in the frequency domain, as in Forni et al (2000 and 2004) have become prominent in the forecasting of GDP given their ability to include unobserved components to predict the future path of GDP.

Fenz, Schneider and Spitzer (2004) made a forecast of Austrian real GDP by combining the forecast of unobserved components and a dynamic factor model. A unique feature of the model is the aggregation procedure used to derive quarterly GDP growth rates. Kalman filter technique was used to estimate and extract the unobserved GDP growth rate series. Basing their estimation on an autoregressive term and exogenous conjugal indicators, the authors noted the short-term out-of-sample forecast of real GDP in Austria performing significantly better than the benchmark models. Similarly, the model accurately predicts the latest data relative to the first data release.

For the US economy, Allen and Pasupathy (1997) adopted the state space representation in forecasting the fiscal and monetary control (exogenous) variable. They specified a linear relation between state variables, allowing for time variation using a recursive least squares with exponential forgetting factors and ordinary least squares estimations. The forecast result show that a one-step-ahead recursive least squares estimates tracks the actual annual monthly GDP growth rates perfectly while its algorithm shows a better performance in the out-of-sample forecast especially for state variables that exhibit the greatest cyclical variation.

Schulz (2007) applied the small scale state space and the large scale static principal components models propounded by Stock and Watson (1991, 2002) to Estonian data to forecast real economic growth and then benchmark the result against other forecasting models. Estimating the model with maximum likelihood and the Kalman filter procedure, the result suggest the relevance of financial data, particularly growth in monetary aggregate and investment and some survey type data, over other economic data in forecasting the Estonian economic trajectory. The model could not, however, identify classical business cycle with booms and recessions in Estonia.

Benkovskis (2008) used coincident information from monthly industrial production, retail trade, broad money aggregate, and confidence indicators to estimate a short-term forecast of Latvia's real GDP growth in a state space representation. The study used quarterly univariate forecasting (bridge) equations incorporated into state space framework containing real GDP and 28 vintage of quarterly real GDP. The result showed that the bridge equations that include broad money aggregates performs better than the ARIMA models even though both models provide valuable information when broad money aggregatesare included.

Recently, one of the most applied real GDP forecasting techniques in analyzing time dependent system has been the dynamic factor models in state space representation owing largely to its flexibility in information extraction (see Anderson and Gascon, 2009; Camacho et al., 2013; and Hindrayanto et al., 2014).

Anderson and Gascon (2009) employed the state space framework in estimating the 'true' unobserved measure of total output in the US economy. The objective (is) was to estimate the 'true' value of real output for use in the construction of trend-like measures of potential output. Using revised statistics with recent data vintage in a state space framework to extract estimates of the 'true' series, the model result suggested an improved real GDP closer to 10 per cent reduction in uncertainty. In terms of forecast accuracy of a state space framework, these results were consistent with those in Baek (2010).

Camacho et al., (2013) adopted a small scale dynamic factor model in forecasting Argentine's real GDP using mixed frequency vintage economic data. The model was able to capture the turning points of the highly volatile Argentine's GDP growth path mimicking the history of the country's business cycle. In addition, the model's precision in producing reliable back-cast and now-cast real data was ascertained as it explained about 89 per cent of the volatility in Argentine's real GDP growth.

In terms of forecast evaluation using the root mean squared error and the mean absolute error, Jovanovic and Petrovska (2010) gauged the forecast ability of six different short-term modelsfor the Macedonian economy including the Kalman filter. The model result indicated that the static factor model outperforms other models, suggesting that models with large data sets improved forecast ability. In a similar study for the Euro area, Hindrayanto et al., (2014) found the dynamic factor models to be superior over other models showing a forecast precision of about 77 per cent in the mean squared error.

The dynamic factor models have been applied in Germany, Australia and Macedonia. There is yet, no evidence of the application of dynamic component model in Africa.

Chapter Three
3.0 Cyclical Properties of Output Growth in Nigeria

To further understand the cyclical properties of output growth the regime switching behaviour is investigated in line with Hamilton (1989) and Schulz (2008). This enabled us disaggregate the state of the economy growth cycles into regimes as per whether growth rate was rising (the state of output acceleration i.e. "growth expansion") or declining (the state of output slowdown i.e. "growth contraction"). Hence, a 2-regime multivariate Markov Chain model is fitted where GDP growth (Hence, a 2-regime multivariate Markov Chain model is fitted where GDP growth ΔGDP is allowed to follow an AR(1) process.[2]

The first regime indicates growth contraction and captures approximately about three cyclical periods in economic activity in the last one and half decade. These are observable for the periods, 2004-06, period of banking crisis and consolidation; 2007-09, the impact of the global economic and financial crises which slowed growth; and for the most part, from 2010 to date, that growth moderated to low growth levels.

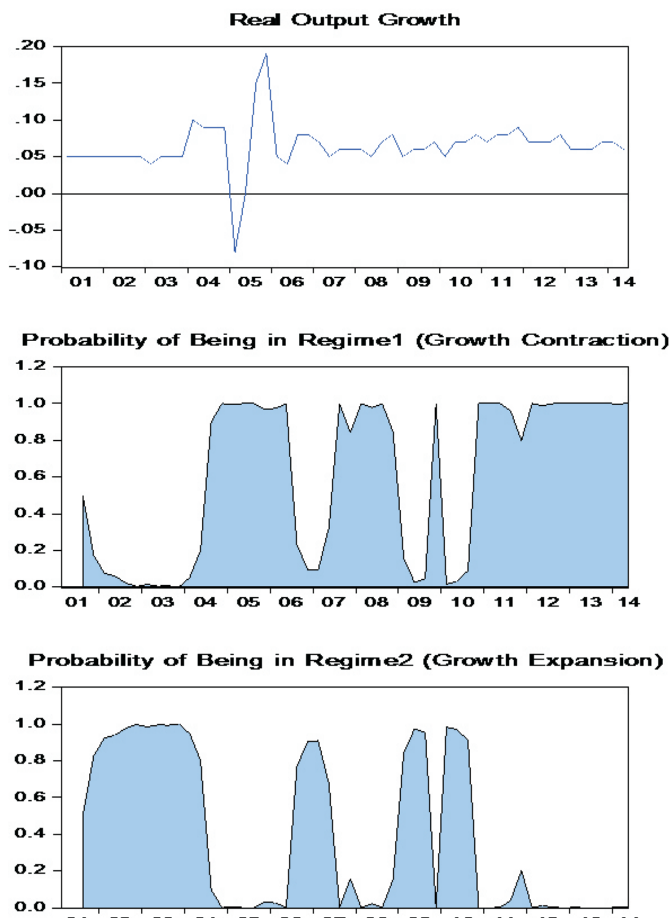
The estimated regime switching intercepts, government revenue, government expenditure, crude oil production and core credit to the private sector, are the drivers in explaining the growth cycles. An analysis of the matrix of transition probabilities shows that the two regimes are relatively not permanent, since the estimated probability is less than one.

Figure 1 characterised Nigeria's growth cycles for the period 2001:Q1 to 2014:Q2 and showed that incidences of retarding

[2] See appendix for the specification of the Markov Chain Model.

growth rate occurred more frequently than of growth expansion. Moreover, as observed in the middle panel that chart, output decelerations seemed to persist for longer periods than growth expansions. While growth contraction does not connote recession (or an absolute decline in output) a sustained deceleration could eventually cause the economy to shrink. The estimated transition probability matrix and the associated ergodic probabilities cyclical states are presented in Table 1.

Figure 1 : Switching State Probabilities for Annualised Real Output Growth Rates



There is an 86.5 per cent probability of remaining in regime 1, a contraction, which is higher than the 78 per cent probability of staying in the expansion state, which implies that the contraction regime is more persistent than expansion regime. Also, the transition probability from a regime of low growth towards a higher growth is 13.5 per cent, lower than the 21.6 per cent probability of exiting a high growth towards lower growth. Further, the ergodic probabilities indicate that on average contraction lasts about 7.4 months, while the higher growth regime continues for approximately 4.6 months, suggesting more periods of growth contraction.

Table 1: Summary Results of the Markov Switching Model

Transition matrices			**Expected duration**		
	1	2		Regime 1	Regime 2
Regime 1	0.865	0.135	All periods	7.39	4.62
Regime 2	0.216	0.784			

Chapter Four

4.0 Methodology

4.1 Determination of Variables for Inclusion in the Dynamic Factor Model

In many cases, macroeconomic variables could exhibit co-movements with phase-shifts across a time domain. Hence, it is important to measure these inter-temporal cross-correlations at different lags and leads. Conventionally, a rolling cross-correlation, termed dynamic correlations, is computed between a reference series and an array of macroeconomic series at predetermined time band. In this study, the dynamic correlation of a reference series x_t other series $y_{i,t-k}$ at lag/lead k is determined as:

$$\rho_{x,y_i}(k) = \frac{Cov(x_t, y_{i,t-k})}{\sqrt{Var(x_t)Var(y_{i,t})}}, \text{for} \quad i = 1,2,\cdots,N; \quad \text{and } k = -6,-5,\cdots,+6$$

The resulting solution gives a scalar quantity, x, at time t and the k^{th} lag/lead of series i . In the immediate discussion, dynamic correlations are determined between GDP growth, as the reference series, and some selected macroeconomic variables over a 6-lead-lag time domain. This provides some information regarding the relative importance of individual variables and the comparative prominence of backward- versus forward-looking expectation.

Figure 1, depicts the charts of the dynamic correlations with reference to GDP. Panel (a) of that figure plots the inter-temporal co-movements of various interest rate variables with GDP growth. The plots showed a generally downward sloping trend from lags to leads. The implication of this is that not only would a contemporaneous rise in interest rates lower output growth, expected increases would decelerate aggregate demand. The co-movements of lagged interest, however, showed mixed results. While policy and lending rates maintained a negative coefficient, deposit rate exhibited a positive value between lags 3 and 6. This is not unexpected, as increases in deposit rates would increase future consumption and aggregate demand[3]. Examination of that chart indicates that interest rate on 3-month deposits correlated more with GDP growth rate, on the average, over the leads and lags while monetary policy rate had the least average coefficient.

The quantity theory of money suggests a positive relationship between money and output, ceteris paribus. In panel (b) this relationship was examined. Monetary aggregates, therein, exhibited wave-like cycles of dynamic correlations, which were predominantly negative; suggesting a largely inverse relationship between monetary gaps and output growth[4]. The chart generally suggested that monetary overshoots are detrimental to economic growth whether in a forward- or backward-looking framework. Comparing leads and lags, the plots indicated that backward adjustments were considerably more relevant for GDP growth, with respect to money. The largest but negative correlations occurred in the second lag. In the contemporaneous, positive co-movements are seen for base money and narrow money, albeit weakly.

Liquidity management actions of the CBN do not only affect banks' ability to create credits but could impact on aggregate demand. This informs the CBN's use of CRR both as a prudential instrument and a monetary policy tool. Panels (c) and (d) showed the dynamic correlations of output growth with bank

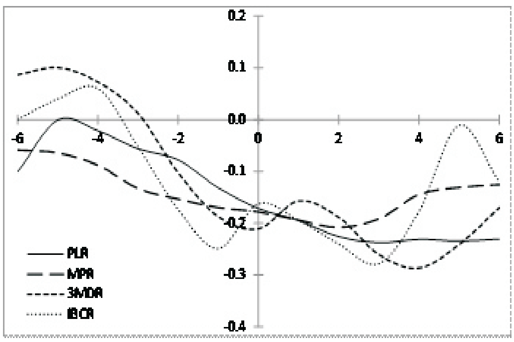
[3] Due to the inter-temporal substitution and income effect in household utility function

[4] Monetary gaps are defined as the difference between monetary growths and the respective monetary growth targets determined via a process of financial programming.

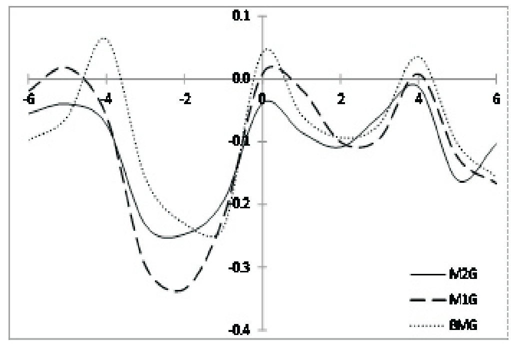
reserves and domestic credits. The relationships plotted in panel (c) suggested that rising reserve requirement could retard economic growth given the negative correlation coefficients over the leads and lags. While the positive coefficients for excess reserve indicate that it could heighten aggregate demand, create excess demand, accelerate output growth and overheat the economy. The chart suggested that past adjustments of the CRR (particularly in the second lag) maintained a slightly larger relationship with contemporaneous GDP growth than did expected adjustments.

Similarly, past outcomes of excess reserve had more relationship with GDP growth, albeit marginally, than expected future outcomes. In panel (d) credits are seen to relate positively with output growth over leads and lags. The dynamic correlations of NDC and CPS trended upward from lag to lead, suggesting that future expectations of credit is more important for output growth than past outcomes of credit. Besides, NDC is seen to generally correlate more with output growth than did credit to private sector. This could be indicative of the overriding importance of government credits to the economy.

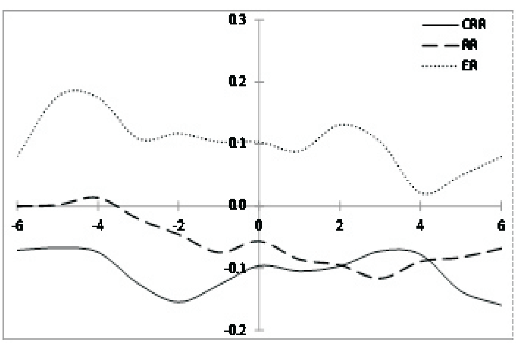
Figure 2 : Dynamic Correlation of GDP growth with Selected Macroeconomic Variables



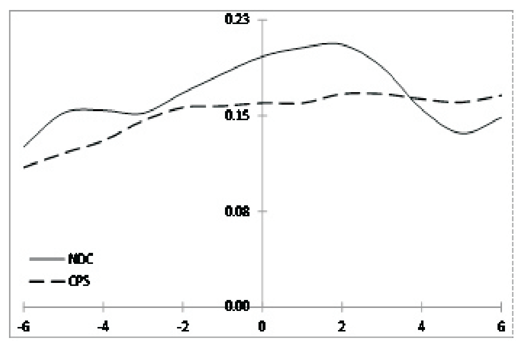
(a) Interest Rates



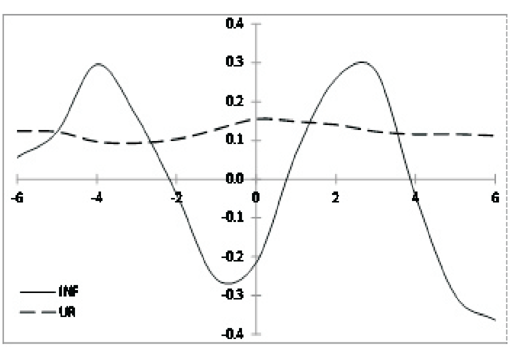
(b) Monetary Aggregates



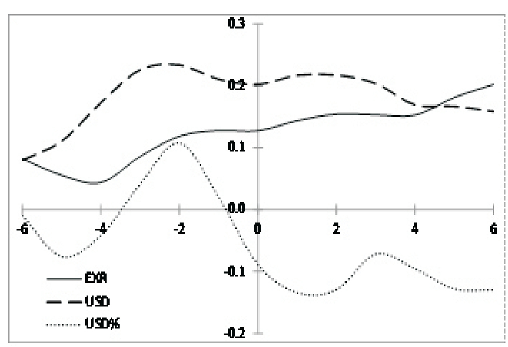
(c) Cash Reserves



(d) Domestic Credits



(e) Phillips Curve Variables



(f) Exchange Market Variables

The plots in panel (e) showed the relationships of GDP growth with inflation and unemployment rates. The dynamic correlations were somewhat perverse of most leads and lags. For instance, unemployment rate could be seen to maintain fairly even positive inter-temporal correlations with GDP growth. Theoretically, economic expansion is posited to diminish unemployment rate; hence, an expected negative relationship is expected. Though the positive sign is a puzzle, it nonetheless highlights the idiosyncratic nature of the Nigerian economy. This peculiarity of the economy is further illustrated in the output-inflation relationship, which is expected to be positive across leads and lags. However, the inflation plot in panel (e) showed a meandering of the dynamic correlation coefficients from positive to negative values over the leads and lags. Negative coefficients could be seen at and around the contemporaneous period, i.e. between lag 2 and lead 1. The depicted inverse relationship is suggestive of the dominance of aggregate supply on contemporaneous and near-term inflation rate. This aggregate supply claim could be reinforced by the positive coefficients recorded after lag 2. In this regard, inflationary outcome relates directly with economic expansion, which suggested that rising prices (via expected profits) encourages production and bolsters aggregate supply. It is not clear, however, whether lags outperformed leads for the inflation correlations.

In panel (f), external reserves could be seen to have a positive and near symmetric dynamic correlation with GDP growth over leads and lags. The level of exchange rate showed an upward sloping positive relationship, indicating the rising importance of expected exchange rate on output growth. However, exchange rate depreciation (USD%) depicted negative coefficients for contemporaneous and lead dynamic correlations. This indicated that sizeable current and expected future depreciations of the naira-dollar exchange rate related inversely with economic expansion. Conversely, past depreciations, between lags 1 and 3, related positively with output growth. Hence, keeping expected depreciation of the exchange rate low could provide output stabilisation.

Next, conducting a similar analysis with transformed and enlarged data series attempts are made to determine the short-term leading indicators for real GDP. A log-transformed real GDP is de-trended using the Hodrick-Prescott (HP) filter to derive the output gap as a new reference series. Cycles of other suitable variables are also derived in this way. The leading indicators are determined from the dynamic correlations of 52 variables with the GDP gap. These are shown in Table 2.

Cross correlation coefficients are compared for the entire series within individual lead/lag. The top six variables with the highest coefficients in each lead/lag of the time domain are highlighted. For instance in the time domain $k = -6$ the six leading indicators are de-trended real personal disposable income (-0.37), de-trended index of manufacturing production (+0.34), de-trended real personal consumption expenditure (-0.33), de-trended savings deposits of banks (+0.28), de-trended government revenue (-0.28) and de-trended government expenditure (-0.19). Thereafter, a latitudinal comparison is conducted on each row to determine which variables appeared in the top six most frequently. These are thus identified as the leading indicators of economic cycles.

Table 2 : Cyclical Correlation of GDP Gap with Some De-trended Macroeconomic Variables

		K													k_{max}	Number
		-6	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**+1**	**+2**	**+3**	**+4**	**+5**	**+6**		
1	ASI_CYC	0.02	0.02	-0.09	-0.12	0.02	0.02	-0.05	-0.09	0.06	0.07	0.00	-0.03	0.10	+6	0
2	BLTL_CYC	0.00	0.04	-0.02	-0.06	-0.06	0.00	-0.04	-0.08	-0.06	0.02	-0.02	-0.06	-0.04	+1	0
3	BM_CYC	-0.02	**-0.36**	0.13	**0.24**	-0.12	**-0.41**	0.16	**0.31**	-0.12	**-0.42**	0.19	**0.36**	-0.12	+3	**6**
4	CCPS_CYC	0.14	-0.10	-0.17	0.10	**0.17**	-0.11	**-0.24**	0.06	**0.14**	-0.12	**-0.22**	0.09	**0.16**	0	**5**
5	CG_CYC	0.01	0.09	0.10	0.01	-0.01	0.03	0.07	0.02	-0.02	0.01	0.04	-0.04	-0.08	-4	0
6	COP_CYC	-0.04	0.13	0.00	**-0.14**	-0.01	**0.17**	0.06	-0.12	0.00	0.17	0.05	-0.11	-0.01	-1, +3	2
7	CPD_CYC	-0.05	**0.33**	-0.03	**-0.16**	-0.07	**0.33**	-0.02	**-0.18**	-0.08	**0.35**	0.00	**-0.19**	-0.12	+3	**6**
8	CPS_CYC	0.14	-0.10	-0.17	0.10	0.16	-0.11	**-0.24**	0.06	0.13	-0.12	**-0.22**	0.10	**0.16**	0	3
9	CRR_CYC	-0.16	0.01	0.16	0.04	-0.13	0.00	0.12	0.01	-0.09	0.02	0.12	0.01	-0.10	-4	0
10	ER_CYC	0.07	-0.08	0.06	0.04	0.02	**-0.17**	0.05	0.09	-0.03	**-0.21**	0.06	0.10	-0.06	+3	2
11	EUR_CYC	0.10	0.01	-0.05	0.04	0.08	0.00	-0.05	0.00	0.05	0.00	-0.09	-0.08	0.00	-6	0
12	EXR_CYC	-0.03	-0.05	-0.09	0.00	-0.01	0.01	0.00	0.03	0.00	0.02	0.04	0.06	0.01	-4	0
13	GBP_CYC	0.03	0.06	-0.02	-0.01	0.06	0.09	-0.02	-0.07	0.03	0.11	-0.02	-0.11	-0.01	+3	0
14	GRV_CYC	**-0.28**	**-0.16**	**0.26**	0.05	**-0.27**	-0.05	**0.29**	0.02	**-0.30**	0.00	**0.33**	0.02	**-0.31**	+4	**8**
15	GXP_CYC	**-0.19**	-0.11	**0.37**	**-0.15**	**-0.20**	-0.04	**0.41**	-0.10	**-0.26**	-0.08	**0.40**	-0.01	**-0.28**	0	**8**
16	HPCI_CYC	-0.15	**0.25**	0.19	-0.11	-0.09	0.16	0.08	-0.12	-0.05	**0.22**	0.07	**-0.18**	-0.13	-5	3
17	IBCR	-0.02	-0.07	0.03	0.04	-0.05	-0.08	0.00	0.02	-0.04	-0.04	0.02	0.05	0.01	-1	0
18	IEP_CYC	0.02	-0.04	-0.08	0.05	0.06	0.01	-0.04	0.05	0.08	0.07	0.01	-0.03	-0.06	-4, +2	0
19	IIP_CYC	0.18	**0.27**	-0.14	**-0.22**	0.14	**0.24**	-0.04	**-0.16**	0.03	0.16	0.07	-0.05	-0.06	-5	**4**
20	IMAP_CYC	**0.34**	0.13	**-0.24**	-0.11	**0.21**	0.10	-0.16	-0.10	0.12	0.09	-0.07	-0.08	0.04	-6	3
21	IMIP_CYC	0.04	0.15	-0.04	-0.13	0.05	0.15	0.06	-0.05	-0.01	0.10	0.10	-0.02	-0.07	-1, -5	0
22	IMP_CYC	0.14	**0.18**	-0.13	**-0.15**	0.05	0.16	-0.06	-0.08	0.08	**0.19**	-0.06	**-0.20**	-0.04	+4	**4**
23	INF_CYC	0.07	-0.03	0.01	0.08	0.05	-0.08	-0.10	0.01	0.06	0.03	0.00	-0.02	-0.07	0	0
24	INF_GAP	0.06	-0.03	0.01	0.07	0.03	-0.08	-0.11	0.00	0.03	0.02	-0.01	-0.02	-0.06	0	0
25	M12DR	0.02	-0.02	-0.01	0.01	0.00	-0.06	-0.08	-0.04	-0.01	-0.04	-0.05	-0.01	0.03	0	0
26	M1_CYC	0.04	-0.10	-0.04	0.07	0.00	**-0.20**	-0.02	**0.13**	-0.02	**-0.20**	0.00	**0.15**	0.01	-1, +3	**4**

Table 2 contained the cross correlations of GDP gap with 52 other macroeconomic variables. The suffix "_cyc" appended to some variables indicates that such variables were log-transformed and de-trended using the HP-filter. Rate variables, like interest rates, were divided by 100 to express them in ratios. Variables with the suffix "_pgap" indicated the deviation of growth rates from their respective targets. The "k_{max}" column denoted the lead/lag with the highest coefficient of dynamic correlation in the time domain for each variable in a particular row; where the sign "+" referred to leads and "–" indicated lags. The column designated "number" represented the number of times a variable in a given row appeared in the top six over the k time domain.

A review of the table suggested that, with 8 occurrences in the top 6 coefficients, the cycles of fiscal variables (government expenditure and revenue) are the leading indicators for economic cycles. Though, correlation coefficients do not portend causality, the results in this case could suggest the prominence of automatic stabilisers in the conduct of fiscal policy in Nigeria. Next to this are components of aggregate demand (personal consumption and personal disposable income) with 7 occurrences each in the top 6. Other leading indicators are base money, crude oil production and credit to the core private sector.

Table 2 (Cont.d)

		K													k_{max}	Number
		-6	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**+1**	**+2**	**+3**	**+4**	**+5**	**+6**		
27	M1_PGAP	0.03	-0.07	-0.09	0.05	0.04	-0.11	-0.09	**0.12**	0.08	-0.08	-0.10	**0.12**	0.09	+1,+5	2
28	M1DR	0.00	-0.05	-0.02	0.03	-0.02	-0.09	-0.06	0.00	-0.05	-0.07	-0.01	0.03	-0.02	-1	0
29	M2_CYC	0.07	0.02	-0.13	-0.02	0.04	-0.06	-0.09	0.01	0.00	-0.05	-0.05	0.02	0.00	-4	0
30	M2_PGAP	-0.04	-0.01	-0.06	-0.02	-0.01	-0.03	-0.04	0.03	0.00	0.01	-0.01	0.04	0.00	-4	0
31	M3DR	0.01	-0.05	-0.01	0.04	-0.02	-0.10	-0.06	0.01	-0.04	-0.07	-0.02	0.03	0.00	-1	0
32	M6DR	-0.03	-0.06	0.00	0.06	-0.04	-0.10	-0.07	0.02	-0.04	-0.07	-0.02	0.04	0.00	-1	0
33	MLR	-0.04	-0.03	0.00	0.02	0.00	-0.04	-0.03	0.02	0.04	-0.02	-0.04	0.03	0.06	+6	0
34	MPR	-0.02	-0.03	0.00	0.01	-0.01	-0.04	0.00	0.03	0.00	-0.03	0.01	0.05	0.03	+5	0
35	NDC_CYC	-0.02	0.12	0.10	-0.05	-0.05	0.06	0.06	-0.06	-0.09	0.03	0.05	-0.09	-0.13	-4	0
36	NDC_PGAP	-0.14	-0.06	0.12	-0.03	-0.16	-0.06	0.14	0.01	-0.13	-0.02	0.16	0.04	-0.12	-2, +4	0
37	NFA_CYC	0.03	-0.13	-0.12	0.04	0.06	-0.04	-0.03	0.07	0.04	-0.02	0.00	0.09	0.06	-5	0
38	PLR	0.00	-0.02	-0.02	0.00	0.02	-0.02	-0.04	-0.02	-0.02	-0.02	-0.03	-0.01	0.01	0	0
39	PSC_CYC	-0.05	-0.01	0.06	-0.01	-0.09	-0.03	0.05	-0.05	**-0.15**	-0.04	0.09	0.00	-0.12	+2	1
40	PSC_PGAP	-0.05	-0.06	-0.02	-0.01	-0.07	-0.04	0.04	0.03	-0.05	-0.01	0.06	0.07	-0.03	-2, +5	0
41	QM_CYC	0.10	0.12	**-0.20**	-0.10	0.10	0.10	-0.14	**-0.13**	0.01	0.11	-0.08	-0.12	-0.01	-4	2
42	RINV_CYC	-0.10	-0.01	0.07	0.06	-0.06	0.04	0.06	0.04	-0.08	0.02	0.07	0.04	-0.11	+6	0
43	RPC_CYC	**-0.33**	0.01	**0.27**	0.02	**-0.35**	0.01	**0.27**	0.02	**-0.34**	0.01	**0.28**	0.01	**-0.37**	+6	**7**
44	RPDI_CYC	**-0.37**	0.01	**0.28**	0.05	**-0.39**	0.02	**0.29**	0.05	**-0.38**	0.01	**0.29**	0.04	**-0.41**	+6	**7**
45	RR_CYC	-0.09	-0.15	0.12	0.07	-0.08	-0.14	0.12	0.08	-0.07	-0.15	0.13	0.10	-0.06	-5, +3	0
46	SD_CYC	**0.22**	-0.07	-0.16	0.01	0.13	-0.08	-0.09	-0.02	0.02	-0.05	-0.04	0.03	0.06	-6	1
47	SDR	0.02	0.08	-0.07	-0.09	-0.01	0.05	-0.07	-0.05	0.01	0.04	-0.08	-0.05	0.07	-3	0
48	TBR	-0.01	-0.07	0.03	0.07	-0.02	-0.10	0.01	0.09	-0.01	-0.09	0.01	0.09	0.03	-1	0
49	TD_CYC	0.10	0.10	-0.16	-0.08	0.07	0.02	-0.13	-0.06	0.03	0.02	-0.08	-0.05	0.03	-4	0
50	UR_CYC	0.08	0.03	0.03	0.00	-0.01	-0.03	0.03	-0.01	-0.04	-0.05	0.00	-0.02	-0.06	-6	0
51	USD_CYC	0.11	0.04	-0.07	0.04	0.13	0.06	-0.08	-0.03	0.04	0.02	-0.11	-0.05	0.02	-6, +4	0
52	XP_CYC	0.02	0.01	0.01	0.01	0.01	0.01	0.00	0.02	0.05	0.01	-0.02	0.00	0.04	+2	0

Note: See appendix for variable definitions

4.2 Estimation Technique

To provide forecasts for the real output growth, this study adopts a dynamic factor model in a state space framework. The use of the framework is motivated by its standard properties, namely, that it allows for the incorporation of unobserved variables to be estimated simultaneously with observed ones; and that it permits the application of Kalman filtration in a recursive manner to analyse the state space with capabilities for generating time-varying parameters, missing data and effective handling of measurement errors.

4.2.1 Kalman Filtration

This provides a multivariate approach to the estimation of unobservable variables by specifying them as function of the observed variables in a state space framework. In this regard, the procedure combines economic theory with time series tools, allowing the unobservable variables to be interpretable. The procedure for implementing Kalman filtration is usually recursive, utilizing maximum likelihood techniques as shown in equations 4.1 and 4.2.

$$\mathbf{y_t} = \mathbf{Z}_t\boldsymbol{\beta}_\mathbf{t} + \mathbf{d_t} + \mathbf{v_t} \qquad \mathbf{v_t} \sim \mathbf{N}(\mathbf{0}, \mathbf{H_t}) \qquad (1)$$

$$\boldsymbol{\beta}_{\mathbf{t}+1} = \mathbf{T}_t\boldsymbol{\beta}_\mathbf{t} + \mathbf{c_t} + \mathbf{R_t}\boldsymbol{\eta}_\mathbf{t} \qquad \boldsymbol{\eta}_\mathbf{t} \sim \mathbf{N}(\mathbf{0}, \mathbf{Q_t}) \qquad (2)$$

Equation 1 is the measurement, observation or signal equation, while equation 2 is the state or transition equation. In the above equations, $p \times 1$, vector, $\mathbf{y_t}$, are the values of the p observed time series at a given time, t. The $p \times 1$ irregular $\mathbf{v_t}$ vector gives p observation disturbances, one for each of the time series in $\mathbf{y_t}$.

The observation disturbances are assumed to have zero means and positive definite unknown variance-covariance structure given by the variance matrix $\mathbf{H_t}$ is of order $p \times p$. The $m \times 1$ state vector $\boldsymbol{\beta}_t$ represents a set of unobserved variables and unknown fixed effects, while the matrix $\mathbf{Z_t}$ of dimension $p \times m$ links the unobservable variables and regression effects of the state vector with the observation vector[5]. Matrix $\mathbf{T_t}$ in equation 2 is the transition matrix of $m \times m$ dimension. The $r \times 1$ vector $\boldsymbol{\eta}_t$ contains the state disturbances with zero means and unknown variances and covariances collected in the matrix $\mathbf{Q_t}$ of dimension $r \times r$. In the above model, $\mathbf{R_t}$, an $m \times r$ matrix, is included in front of the state disturbance term in the model when $\mathbf{Q_t}$ is singular and $r < m$ to make it work with nonsingular $\boldsymbol{\eta}_t$ Durbin and Koopman, 2001; Commandeur and Koopman, 2007; Mergner, 2009). However, $\mathbf{R_t}$ becomes an identity matrix where $r = m$.

In the light of the above, algebraically, $\mathbf{v_t}$ and $\boldsymbol{\eta}_t$ can be shown to be serially uncorrelated and normally distributed as follows:

$$E(\boldsymbol{\eta}_t \boldsymbol{\eta}_\tau') = \begin{cases} \mathbf{Q_t} & for\ t = \tau \\ \mathbf{0} & otherwise \end{cases} \qquad (3)$$

$$E(\;_{\mathbf{t}}\;'_{\boldsymbol{\tau}}) = \begin{cases} \mathbf{Q_t} & for\ t = \tau \\ \mathbf{0} & otherwise \end{cases} \qquad (4)$$

Also, the correlation between the state and observation disturbances is assumed to be zero and independent of the initial state vector $\boldsymbol{\beta}_t$, represented in algebraic terms as:

[5] The $m \times 1$ state vector $\boldsymbol{\beta}_t$ has an initial state vector $\boldsymbol{\beta}_1 \sim N(\boldsymbol{\alpha}_1, \mathbf{P}_1)$ where $\boldsymbol{\alpha}_1$ is the mean vector and $\mathbf{P_1}$ the covariance matrices.

$$E(\boldsymbol{\eta}\mathbf{v}_t') = \mathbf{0}, \quad t = 1,\ldots,T \tag{4.5}$$

$$E(\boldsymbol{\eta}_t\boldsymbol{\beta}_1') = \mathbf{0}, \quad E(\mathbf{v}_t\boldsymbol{\beta}_1') = \mathbf{0}, \quad t = 1,\ldots,T \tag{4.6}$$

It should be noted that the Kalman Filter estimation processes can be optimal under the assumption of normally distributed error schemes. Given that the conditional distributions of the state vector are normal; their full specification is defined by the first two moments which can be computed using Kalman filter approach. Thus, the conditional mean, the first moment, is an efficient estimator containing minimum MSE matrix. Mergner (2009) show that even if the disturbances are not normal, "the Kalman filter is no longer guaranteed to yield the conditional mean of the state vector.Thus, the Kalman filter can still be applied if the normality assumption is relaxed[6].

4.2.2 Growth Dynamic Factor Model

In the light of the above, the empirical dynamic factor model for forecasting GDP is specified to include six measurement equations and two state equations. The included variables are obtained through the examination of the strength of the dynamic correlation between growth and other macroeconomic indicators. The exercise identifies factors with strong co-movement and the possibility of having common factors with growth. This obviously strengthens forecast performance and provides insight on the growth path from available high frequency macroeconomic time series variables.

[6] In the literature estimators that are derived through the maximization of the Gaussian likelihood function with non-normal observations are called quasi-maximum likelihood (QML) estimators (see Hamilton, 1994; Mergner, 2009).

These factors include the growth in money supply (g_m2) , credit to the core private sector (g_ccps), government revenue (g_grv) government expenditure (g_gxp) and crude oil production (g_cpd). The disturbances for each of the measurement equation $var \quad exp$ are specified to follow standard deviations as follows:

$$g\_gdp = {}_1S_1 \quad S_2 g\_gdp_{t \;\; 1} \quad var \quad exp \quad {}_2{}^2 \tag{7}$$

$$g\_m_2 = {}_3S_1 \quad var \quad exp \quad {}_4{}^2 \tag{8}$$

$$g\_ccps = {}_5S_1 \quad var \quad exp \quad {}_6{}^2 \tag{9}$$

$$g\_grv = {}_7S_1 \quad var \quad exp \quad {}_8{}^2 \tag{10}$$

$$g\_gxp = {}_9S_1 \quad var \quad exp \quad {}_{10}{}^2 \tag{11}$$

$$g\_cpd = {}_{11}S_1 \quad var \quad exp \quad {}_{12}{}^2 \tag{12}$$

The state or transition equations are defined by equation (13) and (14). Stock and Watson (1991) have shown that the common factor (S_1) can also be referred to as an index of coincident indicators. Equation (4.14) captures the time-varying measure of persistence inherent in real output growth and is illustrative of periods of both declining and rising growth persistence.

$$S_1 = {}_{13}S_{1t \;\; 1} \quad var \quad 0.5 \tag{13}$$

$$S_2 = {}_{18}S_{2t \;\; 1} \quad var \quad exp(\;\; {}_{19}) \tag{14}$$

4.2.3 Forecasting Growth

In order to forecast the real output growth, the forecast of (S_t) from the above dynamic factor model is generated over the extended horizon. This is then multiplied by the estimate of lambda $_i g\_gdp$ obtained from equation (7) for each period over the forecast horizon. Thereafter, the mean of the rate of growth of GDP is added to these products recalling that the above model is estimated after the included observation variables were demeaned.

Chapter Five

5.0 Empirical Results

In order to identify a 'better' model, four variants of the 6-variable dynamic factor model (DFM) were estimated by providing alternative characterization for the initial prior of the shock processes. Each model included the demeaned growth rates of GDP, money supply, core credit to the private sector, government revenue, government expenditure and crude oil production. In model 1, shock processes were allowed to follow the standard deviation of the variables, while the initial values of the parameters associated with the common factor in each state equation followed scaling of the standard deviations of each of the observed variable to that of the GDP. The common factor (S_1) and prior of 2.2 on its parameter. Model 3 simply varied the prior for S_1 from 2.2 to 0.9. Model 4 included 3 states, S_1, S_2 and an AR (1) stationary process with the random work prior of 0.9 and its prior shock kept at 0.5.

In the light of the above, an evaluation of the in-sample forecast performance of the 4 alternate specifications was conducted based on the predictive strength of the models. The results of the comparison were reported in Table 3. A stronger prediction depends on ability of the common factor (coincident index) to correctly forecast the movement of the current quarter GDP otherwise, the prediction is said to be weak. The evaluation is made for the entire data set, spanning January 2000 to August 2014. Evaluation relied on the performance of the statistical significance of the final state of the common factor, which was dependent on the RMSE and z-Statistic. In this case, the coincident indicator, as obtained from model 1, is more likely to predict the movement in GDP more correctly than the other models since it is significant and has the lowest probability value.

Table 3: Evaluation of theIn-Sample Forecast Performanceof the Models

	Model 1	Model 2	Model 3	Model 4
RMSE	0.8425	0.8365	0.8365	0.7352
z-Statistic	1.9607	-1.9531	1.9532	1.6940
Prob. value	0.0499**	0.0508	0.0508	0.0903

Note: ** significant at 5.0% level

The estimated values for the factor loadings were reported in Table 4. The factor loadings indicate the degree to which variations in each observed variable are correlated with the latent factor, which is the GDP. As observed, the loading factors of all monetary variables were statistically significant, while the fiscal variables and crude oil production had statistically insignificantfactor loadings.

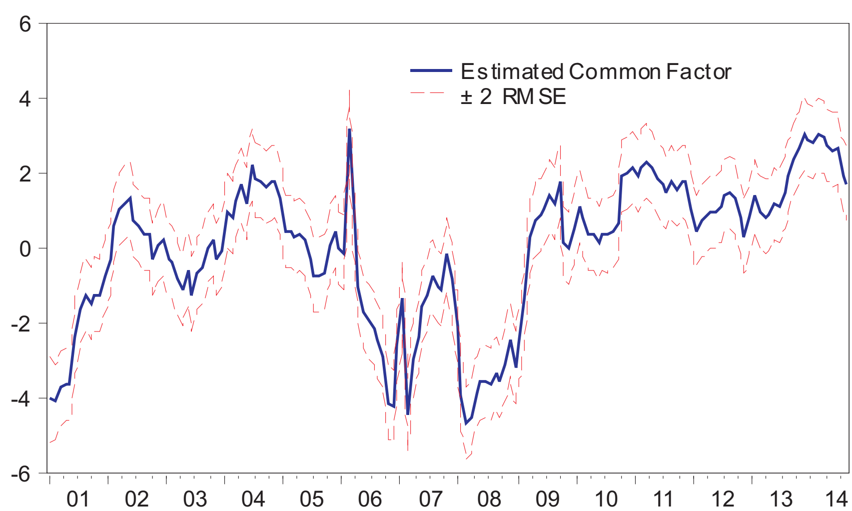
Table 4: Estimated Factor Loadings

	Loadings	Shock
g_GDP	**0.2391**	**1.5340**
	(0.1747)	(0.0000)
g_M2	**-6.7693**	**1.7002**
	(0.0000)	(0.0000)
g_CCPS	**-6.7009**	**2.2729**
	(0.0000)	(0.0000)
g_GRV	**-2.7098**	**2.7025**
	(0.1377)	(0.0000)
g_GXP	**-4.1198**	**2.8767**
	(0.1393)	(0.0000)
g_CPD	**0.4792**	**2.1508**
	(0.3668)	(0.0000)
State_Persistence	**0.9417**	**n.a.**

Note: Figures in parenthesis are p-values

From the estimated regression, the latent dynamic factor was derived from the common component of suite of the six variables included. These observable variables represented underlying driver of economic activities in Nigeria. Generally, the underlying common factor is expected to predict the turning points in economic activities fairly accurately. To derive GDP growth forecast, the dynamic common factor was multiplied by an estimated factor loading and the product was added to the mean of the GDP growth rate over the forecast horizon. The estimated root mean squared error (RMSE) of 0.84 provided a predicted final state which was significant at 5 per cent level. The implication is that the composite common factor was contained within an acceptable confidence band as shown in figure 3.

Figure 3 : Filtered State Estimate of the Common Factor



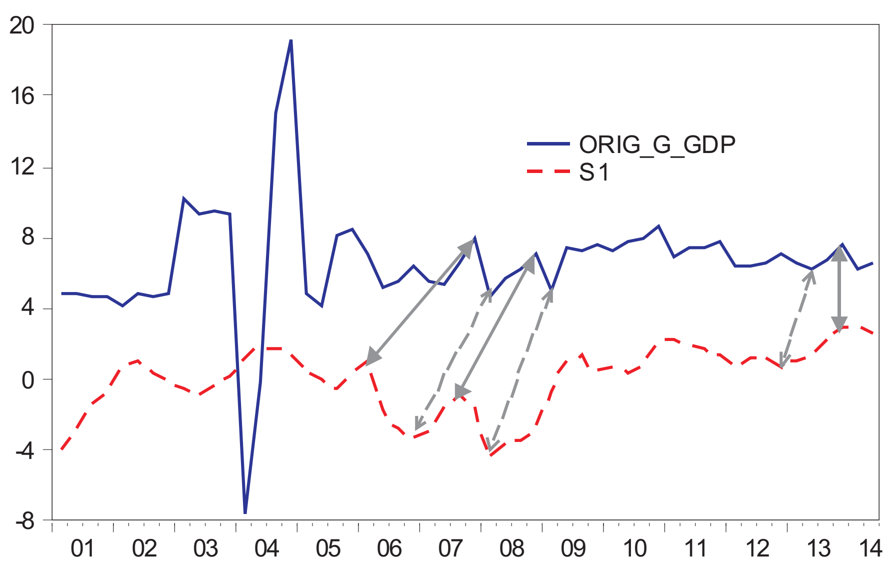
The plot of the unobserved common factor of the included variables exhibited three prominent phases (2001-2006), (2006-2009) and (2009-2014). While the first phase showed a rise with twin spikes in 2002 and 2004, the second phase witnessed moderate declines. The upward beat was resumed in the last sub-sample and that was sustained from 2009 through 2014. A rise (or a fall) in the underlying factor suggested a rise (or a fall) in the GDP rate, depending on the sign (positive or negative) of the loading factors.

5.1 Forecasting the GDP Growth: the Estimated Common Component

The estimated common component, the underlying factor, contained vital information for forecasting economic growth in Nigeria. Many authors including Schulz (2008), Forni et al. (2000), Stock and Watson (2002) and Banerjee et al. (2006) recognised the significance of common components as leading factors for forecasting economic activities. Schulz (2008) using a composite lead derived from dynamic principal component analysis to forecast economic growth for Estonia. In this study, the common factor is derived directly from a dynamic state space specification as the composite lead for GDP growth in a data set that also included money supply, credit to core private sector, government revenue, government expenditure, and Nigeria's crude oil production. The importance of composite lead is not only in forecasting accurate GDP growth numbers, but in its ability to predict turning points which is of considerable value to policymakers (Chin et al., 2000). Figure 4 juxtaposed the estimated dynamic common factor – composite leading indicator – with real GDP growth in Nigeria. It showed a common long-run trend between the underlying factor and the economic growth.

Apart from the outliner growth periods in 2004, the chart depicted some congruence in between the indictor and the reference series. This however, contained phase shifts in the ability of the composite leading indicator to track turning points in real GDP growth. A fall in the reference series is preceded by a decline in the composite leading indicator, and vice versa for a rise. However, the number of lags (i.e. delay) between the turning points in the indicator and reference series seem to be reducing. This phase shifts suggest improving track-ability of the composite leading indicator.

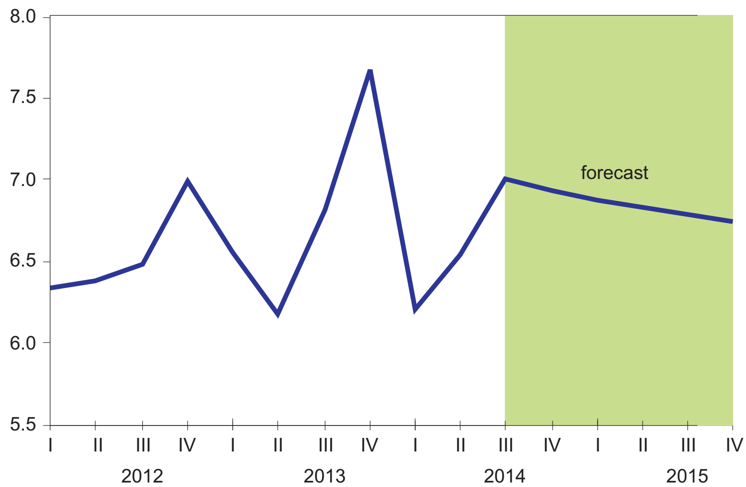
Figure 4 : Composite Leading Indicator and GDP Growth



As shown in the chart, the peaks and trough observed in the real GDP growth around 2007 and 2008 were reliably predicted by composite leading indicator with a delay of nearly 2 years. By 2010, the delay could be seen to have shortened to just over 4 quarters. Towards the end of the plots, the graph indicated that, more recently, delay in predicting the turning points have been reduced to about 3 quarters. Hence, the slightly southward orientation of the composite leading indicator observable towards the end of the series could suggest some deceleration of economic activities by the middle of 2015. This is indeed not inexplicable. The chart showed a plunge in observed real GDP growth rates in the periods after general elections in 2003, 2007, and 2011.

Applying the estimated loading factor for the reference series to the composite leading indicator, and adjusting for the mean, real GDP growth forecasts were derived for the period 2014:Q3 to 2105:Q4. As noted earlier, a RMSE analysis selected the originating state space specification as the best model among alternative specifications. The actual and forecast growth rates were plotted in Figure 2. The forecast showed a robust real GDP growth rate which generally remained above 6.50 per cent. Output acceleration was forecasted for 2014:Q3, with growth rate predicted to rise from 6.54 per cent in 2014:Q2 to 6.93 per cent. Nonetheless, a modest slowdown is predicted towards the end of 2014 and throughout 2015. Based on the composite leading indicator, GDP growth rates for 2015:Q2 and 2015:Q4 were forecasted as 6.83 per cent and 6.75 per cent, respectively, indicative of a slowdown.

Figure 5 : GDP Growth Rate - Actual and Forecast (%)



Chapter Six

6.0 Conclusion

The need for a more consistent and accurate GDP forecast for the conduct of forward-looking monetary policy is quite fundamental. This is because the availability of real-time data is crucial to determine the initial conditions of economic activity on latent variables such as the output gap to make policy decisions.

The study developed a framework that enables the forecasting of quarterly GDP growth for Nigeria using a dynamic factor model in a state space methodology. This approach is anchored on its ability to extract underlining factors among a set of macroeconomic variables that contain information about the latent state of the economy that can be used in generating robust and reliable GDP forecast. The DFM provide help to addressing the paucity of real-time data. It also permits the utilisation of a mixture of data frequencies in the determination of the current state of the economy.

Prior to estimation, the dynamic correlations between the GDP growth rates and fifty-two (52) macroeconomic variables were constructed. This helps in the identification of the best six leading indicators of economic/growth cycles namely; that government revenue, government expenditure, crude oil production and core credit to the private sector. Efforts were also made to characterise this growth cycles for enhanced understanding of the boom and bust sessions associated with business cycles in economic literature. The underlying common factor mimics the turning points in growth trajectory relatively well. It signals three distinct regimes that align with the direction of the movement of GDP growth rate based on the loading factors.

The study revealed that of the four variants of the model investigated, model one (1) appeared to be the most preferred, as it predicted the movement in real GDP growth rate more accurately than the other models. The model showed robust real GDP growth forecasts of 6.54, 6.93, 6.83 and 6.75 per cent in 2014Q3, 2014Q4, 2015Q2 and 2015Q4, respectively. The model did not only assist in computing a coincident lead indicator for Nigeria, but also explained a very high percentage of the variance of actual GDP growth rate. The model is, thus, a valid tool for tracking the business cycle.

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Appendix 1: Variable Definitions

S/N	Series Acronym	Series Name	Weight	Unit
1	ASI	All Share Index		(1984 = 100)
2	BLTL	Bank Loan: Total		Million Naira
3	BM	Base Money		Million Naira
4	CCPS	Credit to Core Private Sector		Million Naira
5	CG	Credit to Government		Million Naira
6	COP	Crude Oil Price		US Dollars/Barrel
7	CPD	Crude Production		Million Barrels/Day
8	CPS	Credit to Private Sector		Million Naira
9	CRR	Cash Reserved Ratio		(%)
10	ER	Excess Reserves		Million Naira
11	EUR	Naira to Euro Rate		(N/€1.00)
12	EXR	External Reserves		Million Dollars
13	GBP	Naira to Pounds Rate		(N/£1.00)
14	GRV	Govt Revenue		Million Naira
15	GXP	Govt Expenditure		Million Naira
16	HCPI	All Items	1000.00	(Nov. 2009 = 100)
17	IBCR	Interbank Call Rate		(%)
18	IEP	Index of Electricity Production		(1990 = 100)
19	IIP	Index of Industrial Production		(1990 = 100)
20	IMAP	Index of Manufacturing Production		(1990 = 100)
21	IMIP	Index of Mining Production		(1990 = 100)
22	IMP	Imports (CIF)		Million Naira
23	INF	Inflation Rate		(%)
24	M12DR	12-Month Deposit Rate		(%)
25	M1	Narrow Money Stock		Million Naira
26	M1DR	1-Month Deposit Rate		(%)
27	M2	Broad Money Stock		Million Naira
28	M3DR	3-Month Deposit Rate		(%)
29	M6DR	6-Month Deposit Rate		(%)
30	MLR	Maximum Lending Rate		(%)
31	MPR	Monetary Policy Rate		(%)
32	NDC	Net Domestic Credit		Million Naira
33	NFA	Net Foreign Assets		Million Naira
34	PLR	Prime Lending Rate		(%)
35	PSC	Private Sector Credit		Million Naira
36	QM	Quasi Money		
37	RINV	Real Investment		Million Naira
38	RPC	Real Personal Consumption Expenditure		Million Naira
39	RPDI	Real Personal Disposable Income		Million Naira
40	RR	Required Reserves		Million Naira
41	SD	Saving Deposit of Banks		Million Naira
42	SDR	Savings Rate		(%)
43	TBR	91-Day Treasury Bill Rate		(%)
44	TD	Total Deposit of Banks		Million Naira
45	UR	Unemployment Rate		(%)
46	USD	Naira to US-Dollar Rate		(N/US$1.00)
47	EXP	Exports(FOB)		Million Naira

Appendix 2: The Markov Chain Model

GDP growth rate is modelled to be characterised by two states: high and low growth periods. The high state represents periods of rising growth rate (output acceleration) and the low state reflects a regime of declining growth rate (output slowdown). A 2-regime multivariate Markov Chain model is, thus, fitted where GDP growth $\Delta\mathrm{GDF}$ is allowed to follow an AR(1) process specified as:

$$\Delta GDP_t = \alpha_0(s_t) + \alpha_1(s_t)\Delta GDP_{t-1} + \gamma \boldsymbol{X}_t + e_t \qquad (3.1)$$

$$s_t = \begin{cases} \text{regime } 1, & \Delta^2\mathrm{GDP} \leq 0; \quad \text{growth contraction} \\ \text{regime } 2, & \Delta^2\mathrm{GDP} > 0; \quad \textit{growth expansion} \end{cases}$$

where $\mathbf{X}_t$ is the $\mathrm{N} \times 1$ vector of exogenised drivers of the growth cycle. As defined by Hamilton (1989) the transition probabilities $p_{21} = P(s_t = 1 | s_t = 2)$ reflects the switch from regime 2 to regime 1, and $p_{12} = P(s_t = 2 | s_t = 1)$ is the probability of switching from regime 1 to regime 2. The associated transition matrix is defined as

$$\mathbf{M} \quad \begin{matrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{matrix} \qquad (3.2)$$

where the conditions $p_{11} < 1$, and $p_{22} < 1$ are satisfied to ensure that the estimated probabilities are irreducible in the 2 states while the ergodic condition $p_{11} + p_{22} > 0$ are also expected to hold. Hence, $p_{11} + p_{12} = 1$ and $p_{21} + p_{22} = 1$ is generally satisfied.

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